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THERMAL RADIATION PHENOMENA

Radiation Hydrodynamics of High Temperature Air

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ABSTRACT

This report introduces the reader to radiation hydrodynamics (RH) and discusses its application to fireballs in the atmosphere. After formulating the basic equations of RH, special attention is given to the radiative transfer problem. Several methods for solving the equations of transfer are touched upon but special emphasis is placed on the two stream method with a frequency averaging procedure, which is specifically designed for use with finite zone sizes. A version of the FIREBALL code which utilizes this approach is described. The physics of fireballs is illustrated with the example of a one kiloton detonation at sea level density and without interference from the ground. Some remarks are made on scaling procedures for extending the results to higher yields and altitudes. Estimates are made of the validity of the models.

FOREWORD

"Thermal radiation" is electromagnetic radiation emitted by matter in a state of thermal excitation. The energy density of such radiation in an enclosure at constant temperature is given by the well known Planck formula. The importance of thermal radiation in physical problems increases as the temperature is raised. At moderate temperatures (say, thousands of degrees Kelvin) its role is primarily one of transmitting energy; whereas at high temperatures (say, millions of degrees Kelvin) the energy density of the radiation field itself becomes important as well. If thermal radiation must be considered explicitly in a problem, the radiative properties of the matter must be known. In the simplest order of approximation, it can be assumed that the matter is in thermodynamic equilibrium "locally" (a condition called local thermodynamic equilibrium, or LTE), and all of the necessary radiative properties can be defined, at least in principle. Of course whenever thermal radiation must be considered, the medium which contains it inevitably has pressure and density gradients and the treatment requires the use of hydrodynamics. Hydrodynamics with explicit consideration of thermal radiation is called "radiation hydrodynamics".

In the past twenty years or so, many radiation hydrodynamic problems involving air have been studied. In this work a great deal of effort has gone into calculations of the equilibrium properties of air. Both thermodynamic and radiative properties have been calculated. It has been generally believed that the basic theory is well enough understood that such calculations yield valid results, and the limited experimental checks which are possible seem to support this hypothesis. The advantage of having sets of tables which are entirely calculated is evident: the calculated quantities are self-consistent on the basis of some set of assumptions, and they can later be improved if calculational techniques are improved, or if better assumptions can be made.

The origin of this set of books was in the desire of a number of persons interested in the radiation hydrodynamics of air to have a good source of reliable information on basic air properties. A series of books dealing with both theoretical and practical aspects was envisaged. As the series materialized, it was thought appropriate to devote the first three volumes to the equilibrium properties of air. They are:

The Equilibrium Thermodynamic Properties of Air,
by F. R. Gilmore

The Radiative Properties of Heated Air,
by B. H. Armstrong and R. W. Nicholls

Tables of Radiative Properties of Air,
by Lockheed Staff

The first volume contains a set of tables along with a detailed discussion of the basic models and techniques used for their computation. Because of the size of the related radiative tables and text, two volumes were considered necessary. The first contains the text, and the second the tables. It is hoped that these volumes will be widely useful, but because of the emphasis on very high temperatures it is clear that they will be most attractive to those concerned with nuclear weapons phenomenology, reentry vehicles, etc.

Our understanding of kinetic phenomena, long known to be important and at present in a state of rapid growth, is not as easy to assess as are equilibrium properties. Severe limitations had to be placed on choice of material. The fourth volume is devoted to general aspects of this topic. It is:

Excitation and Non Equilibrium Phenomena in Air,
by Landshoff, et al.

It provides material on the more important processes involved in the excitation of air, criteria for the validity of LTE and special radiative effects.

A discussion of radiation hydrodynamics was felt to be necessary and the fifth volume which deals with this topic is:

> Radiation Hydrodynamics of High Temperature Air,
> by Landshoff, Hillendahl, et al.

It reviews the basic theory of radiation hydrodynamics and discusses the application to fireballs in the atmosphere.

The choice of material for these last two volumes was made with an eye to the needs of the principal users of the other three volumes.

Most of the work on which these volumes are based was supported by the United States Government through various agencies of the Defense Department and the Atomic Energy Commission. The actual preparation of the volumes was largely supported by the Defense Atomic Support Agency.

We are indebted to many authors and organizations for assistance and we gratefully acknowledge their cooperation. We are particularly grateful to the RAND Corporation for permission to use works of F. R. Gilmore and H. L. Brode and to the IBM Corporation for permission to use some of the work of B. H. Armstrong. Most of the other authors are employed by the Lockheed Missiles and Space Company, in some cases as consultants.

Finally, we would like to acknowledge the key role of Dr. R. E. Meyerott of LMSC in all of this effort, from the initial conception to its realization. We are particularly grateful to him for his constant advice and encouragement.

Criticism and constructive suggestions are invited from all readers of these books. We understand that much remains to be done in this field, and we hope that the efforts represented by this work will be a stimulus to its development.

The Editors

J. L. Magee
H. Aroeste
R. K. M. Landshoff

Preface

This volume reviews the basic theory of radiation hydrodynamics and discusses the application to fireballs in the atmosphere. The first chapter starts with a formulation of the basic equations and goes on to discuss schemes for translating these impossibly difficult equations into manageable computing procedures. As a companion to this chapter we have added Appendix A with a version of Hillendahl's FIREBALL code, which runs without inputs of a classified nature.

Chapter 2 deals with the physics of fireballs. The main discussion is devoted to the description of a one kiloton detonation at sea level. That section has nearly all been written by H. L. Brode of the RAND Corporation but a few passages have been added by the editor. One of these deals with opaque precursors to shocks whose significance to the thermal output was noted by Hillendahl since the original version was written. The section on other yield and altitudes was also written by the editor.

The summary chapter examines the reliability of the results and how this is affected by approximations, incomplete basic information and other deficiencies in the present state of the art.

I would like to thank Dr. H. L. Brode for his contribution and the RAND Corporation for permission to include his work in this volume.

R. K. M. Landshoff

Contents

Chapter 1. RADIATION HYDRODYNAMICS

1.1 Introduction

A nuclear detonation deposits a large amount of heat energy in the air around it. The heating phase is of relatively short duration since the energy arrives in the form of X-rays which come either directly from the surface of the exploding bomb or from the shock heated air in the immediate vicinity of that surface.

Following the X-ray deposition the air approaches local thermodynamic equilibrium (LTE). The method of calculating the subsequent explosion history which is discussed in this chapter ignores this period where the air relaxes to LTE. Before we proceed we take a short look at the validity of that assumption.

The kinetics of relaxation processes has been discussed in Chapter 6, (4)*. The relaxation time depends on the ambient air density and on the final temperature as shown in Fig. 6.1 (4).

For a detonation at sea level practically all the energy deposited by X-rays gets stuck in a relatively small volume and raises the temperature to very high values. Under these conditions relaxation times are very short. For a detonation at a high altitude a sizeable fraction of the X-ray energy is deposited at large distances and produces a lesser temperature rise because of the inverse square drop of the flux density. The lower air density and the lower temperature both contribute to increase the relaxation time.

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* DASA-1917-4, from now on referred to as (4).

As an example, let us consider a detonation with an X-ray yield of 10^{20} ergs radiating like a blackbody with a temperature of 10^7 ^oK occurring at an altitude somewhat below 50 km where the air density is 10^{-3} times less than at sea level. A crude estimate, using the asymptotic theory of Section 4.4, (4) shows that about 10% of the X-ray energy is deposited at distances more than about 80 m where it produces temperatures less than 12,000^oK. In Fig. 6.1 (4) one reads off that the relaxation time at that temperature and a density $\frac{\rho}{\rho_o} = 10^{-3}$ is 10^{-6} sec. Within that sphere it takes less time and on the outside more time to relax the air to its equilibrium temperature. Thus 10% of the energy relaxes at a relatively slow rate and the assumption that one can ignore the relaxation period is not entirely justified in that case.

The assumption of LTE is essential to the classical formulation of hydrodynamics. It means that the temperature is a well defined property of the fluid and that pressure and internal energy are known functions of density and temperature. Without LTE it would be much more difficult to formulate the conservation theorems for momentum and energy.

In the theory of radiative transfer (Chapter 2, (2)*, which together with hydrodynamics accounts for the expansion of fireballs, LTE also is an assumption of major importance. Without it a quantitative prediction of the interaction between matter and radiation would be a hopelessly complicated problem.

Despite the very important role played by radiative transport radiation does not as a rule account for a significant fraction of the energy density and the pressure within a fireball. Even for blackbody radiation,

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* DASA 1917-2, from now on referred to as (2).

which is not present unless the gas is opaque, this contribution is small unless the temperature exceeds values like 25 eV. Temperatures of that magnitude are only maintained during the very early stages of fireball histories. In this period the fireball cools down by radiative expansion and this goes so fast that there is essentially no hydrodynamical motion. In formulating the hydrodynamic equations one can therefore ignore the energy density and pressure of radiation because by the time they get into the act they are indeed negligible.

During the early period of fireball expansion (where radiative transfer of energy is important) the shape generally appears to be almost spherical, at least at low altitudes where the size is small compared to the scale height of the atmosphere. Asymmetries which are hidden by the opaque outer layers may possibly occur due to instabilities at the bomb air interface, but we shall ignore these. Not much is known about such phenomena in any case and adding the complication of asymmetry would compromise the already complicated problem of treating radiation flow. In line with the current state-of-the-art we shall therefore discuss only spherically symmetrical problems.

1.2 Basic equations of radiation hydrodynamics

The differential equations for calculating fireball histories are the conservation relations of ordinary hydrodynamics but with a rather complicated heating term in the energy equation. They can be written in either Eulerian or Lagrangian form. The two forms are characterized by a different choice of independent space variables. In the Eulerian system these are the coordinates in real space and in the

Lagrangian one they are coordinates which are tied to the particles of the fluid. In the Lagrangian system the coordinates in real space which describe the position of a specified particle are used as dependent variables. In the Eulerian system this is manifestly impossible and the motion is described in terms of the fluid velocity.

The other dependent variables which characterize the thermodynamic state of the fluid are the same in the two systems and can be chosen from a set which includes the density ρ or its reciprocal the specific volume V, the pressure p, the temperature T, the internal energy E, etc. It may be convenient to keep several of these variables in the equations but one must keep in mind that they are interrelated and that the thermodynamic state is specified by any two of them.

The Lagrangian method is especially useful in problems with a high degree of symmetry where one needs only one coordinate to specify the position. Having restricted ourselves to spherically symmetrical problems we shall therefore adopt the Lagrangian approach.

We define the Lagrangian radius r of a given particle as its radius at time zero, i.e. before it has started to move. The actual radius of the particle at any time is denoted by the capital letter R. The hydrodynamic problem is to find $R(r,t)$.

If ρ_o stands for the initial density the specific volume at any instant is

$$V = \frac{1}{\rho_o}\left(\frac{R}{r}\right)^2 \frac{\partial R}{\partial r} \qquad \text{(conservation of mass)} \qquad (1.2\text{-}1)$$

Introducing the velocity

$$u = \frac{\partial R}{\partial t} \tag{1.2-2}$$

the other conservation equations are

$$\frac{\partial u}{\partial t} = - \frac{1}{\rho_o} \left(\frac{R}{r}\right)^2 \frac{\partial p}{\partial r} \quad \text{(conservation of momentum)} \tag{1.2-3}$$

$$\frac{\partial E}{\partial t} + p \frac{\partial V}{\partial t} = V \dot{Q} \quad \text{(conservation of energy)} \tag{1.2-4}$$

where the rate of heating per unit volume $\dot{Q}$ still needs to be worked out.

As it stands the energy equation has a serious defect because it does not allow for the entropy raise produced by a shock. To get around this we adopt the method of Von Neumann and Richtmyer (1950) and add a pseudo-viscous pressure

$$q = \begin{cases} (\ell^2/V) \ (\partial u/\partial r) & \text{if } \frac{\partial u}{\partial r} < 0 \\ 0 & \text{if } \frac{\partial u}{\partial r} > 0 \end{cases} \tag{1.2-5}$$

to the regular pressure in Eqs. (1.2-3) and (1.2-4). The constant ℓ has the dimensions of a length; it will be further specified when we go to finite difference equations.

The radiative heating rate $\dot{Q}$ at some point is the difference between absorbed and emitted power per unit volume

$$\dot{Q} = \iint \mu'_\nu \ (I_\nu - B_\nu) \ d\nu \ d\Omega \tag{1.2-6}$$

The emitted power presents no problem because the blackbody intensity B_ν is a known function of temperature. There is no angular dependence and the integral over frequency can be expressed in terms of the Planck mean, Eq. (2.4-15), (2). One obtains:

$$\dot{Q}_{em} = 4\bar{\mu}_p \sigma T^4 \tag{1.2-7}$$

where σ is the Stefan Boltzmann constant.

The absorbed power is much more difficult to evaluate because the calculation of the intensity I_ν is a major task. To carry this out one should in principle solve the equation of transfer (Eq. (1.3-1)) along every ray passing through the point in question and for all values of the frequency. One of the major difficulties of such a program arises from the fact that the optical properties of air in a large part of the relevant temperature range result mainly from transitions between molecular levels. The spectrum associated with the major band systems consists of an enormous number of lines and the absorption coefficient fluctuates from large values at the line centers to small ones between the lines. Because of these "windows" the radiation at some point generally comes from points along the ray which are an appreciable distance further back. This distance varies just as strongly with frequency as μ_ν' itself and it is therefore not proper to use local averages of μ_ν' in a frequency interval containing, say, a few lines. Instead, it is in principle necessary to integrate the transport equation at a large enough number of frequencies within every one of these intervals. This correct approach clearly demands an impossible amount of computational

effort which has to be avoided. There are two limiting situations where this can be readily done. The one situation arises for a transparent medium where the optical depth $\mu_\nu' L$ (L being the size of the radiating region) is uniformly small compared to unity. In that case I_ν is very much smaller than B_ν and one can neglect the absorption altogether. In that case we have $\dot{Q} = - \dot{Q}_{em}$ which we know from Eq. (1.2-7).

In the opposite extreme of an opaque region for which $\mu_\nu' L \gg 1$ one can simplify Eq. (1.2-6) directly. The heating rate can in that case be expressed in the form*

$$\dot{Q} = - \vec{\nabla} \cdot \vec{F} \qquad (1.2\text{-}8)$$

The flux vector is given by Eq. (2.5-8), (2) which we rewrite in the form

$$\vec{F} = - \frac{4}{3} \lambda_R \sigma \vec{\nabla} T^4 \qquad (1.2\text{-}9)$$

Tables and graphs of the Rosseland mean free path λ_R or the related opacity can be found in (3)**, p. 12, pp. 446 to 449 and pp. 622 to 625.

The above method of treating radiative transfer was originally developed by Eddington nearly half a century ago. In its application it was however, limited to astrophysical problems where it was not coupled to hydrodynamics. An early discussion of the use of this so-called diffusion approximation to radiation hydrodynamics has been given by Magee and Hirschfelder (1953). The first calculations carried out with this method to appear in the open literature were presented by Marshak (1958).

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* The operator $\vec{\nabla}$ is defined in Eulerian space. In plane or spherical geometry it is well known how to express it in Lagrangian form.

** DASA 1917-3.

1.3 Average absorption coefficients

In the temperature range where molecular transitions occur and where optical depths are neither uniformly small nor uniformly large one has to resort to approximation schemes. It is clearly necessary to apply some kind of frequency averaging which will do a fair amount of violence to the "correct approach" of solving the transfer equation for a few million values of the frequency. The basic mathematical problem is that one wants to average the product $\mu_\nu' I_\nu$ which enters in Eq. (1.2-6) as well as in the transport equation*

$$\frac{dI_\nu}{ds} = \mu_\nu' (B_\nu - I_\nu) \qquad (1.3\text{-}1)$$

by equating the average of the product and the product of the averages, i.e. one wants to replace $\overline{\mu_\nu' I_\nu}$ by $\overline{\mu_\nu'}\ \overline{I_\nu}$ and that is of course not correct. The quality of this approximation depends on the amount of fluctuation among the values of μ_ν' and I_ν that are being averaged and in a line spectrum this fluctuation may be quite severe. A number of averaging schemes have been proposed and are used in various computing programs.

One scheme divides the spectrum into groups (10 to 100) whose widths are chosen fairly narrow at the low energy end and wider as the energy goes up. Within each interval a Rosseland type average is obtained. Such group averages have been used, for example, in the SPUTTER program of AWFL as reported in RTD-TDR-63-3128 Vol. II and in a code developed by J. Zinn of LASL.

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* Strictly speaking, the left hand side of this equation should contain the additional term $\frac{1}{c}\frac{\partial I_\nu}{\partial t}$ but because of the large value of the light velocity this time dependence is usually left out. We note further that light rays are straight lines in Eulerian space and in this section we temporarily abandon the Lagrangian system.

A second method of averaging uses the average transmission function (Eq. (2.6-12b), (2)

$$\overline{Tr\,(\mu'_\nu s)} = \frac{1}{\Delta\nu_i}\int_{\Delta\nu_i} e^{-\mu'_\nu s}\, d\nu \qquad (1.3\text{-}2)$$

and the slab absorption coefficient related to it by Eq. (2.6-19), (2)

$$\overline{\mu_\nu}(s) = -\frac{1}{s}\ln \overline{Tr\,(\mu_\nu' s)} \qquad (1.3\text{-}3)$$

These averages are defined for slabs of thickness s in which the temperature and density of the air are uniform. The intervals $\Delta\nu_i$ are much narrower than the groups of the first mentioned method. The spacing between intervals is 10 to 20 times as large as the interval size. The calculated averages depend smoothly on the frequency so that it seems reasonable to interpolate. The slab average is made to order for use in finite difference equations where the fluid is divided into zones. It has been used in a number of LMSC codes which will be discussed later in this chapter.

A variation of the group average procedure consists of subdividing the frequencies within a group into subgroups which are ordered according to the magnitude of the absorption coefficient rather than by frequency. The spread between the absorption coefficients within each subgroup is obviously less than between those in the entire group and subgroup averages

will therefore be more meaningful. To use subgroup averages we must also introduce individual intensities for each subgroup. Even the use of only two subgroups would improve the accuracy considerably. A short-cut for the calculation of two subgroup absorption coefficients consists of fitting the average transmission function in the form (Eq. (2.6-46), (2).

$$\overline{Tr\,(\mu_\nu's)} = \frac{1}{2}\,(e^{-\mu_1' s} + e^{-\mu_2' s}) \qquad (1.3\text{-}4)$$

Tables of μ_1' and μ_2' are given by Churchill et al. (1963). We don't know of any code which has utilized this type of average.

1.4 Solution of the equation of transfer

Having obtained an average absorption coefficient which permits us to replace the average product $\overline{\mu_\nu I_\nu}$ by the product of averages $\overline{\mu_\nu}\,\overline{I_\nu}$ the transfer equation becomes*

$$\frac{dI_\nu}{ds} = \overline{\mu_\nu}\,(\bar{B}_\nu - \bar{I}_\nu) \qquad (1.4\text{-}1)$$

The formal integration of this equation along a ray is straightforward and leads to

$$\overline{I_\nu}(s) = e^{-\tau_\nu}\,\overline{I_\nu}(s_o) + \int_o^{\tau_\nu} \overline{B}_\nu(s')\, e^{(\tau_\nu' - \tau_\nu)}\, d\tau_\nu' \qquad (1.4\text{-}2)$$

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* The absorption coefficient is still meant to include the correction factor for induced emission (Eq. (2.2-11a), (2) and the prime is left out for convenience of writing only.

$$\tau_\nu = \tau_\nu(s) = \int_{s_o}^{s} \bar{\mu}_\nu(s_1)\, ds_1 \quad ; \quad \tau_\nu' = \tau_\nu(s') \tag{1.4-3}$$

The difficulty arises because one has to determine the value of this integral for all the rays through a given point to evaluate the rate of absorption of radiative energy at that point. This is required for carrying out the angular integration in Eq. (1.2-6).

There are basically two approaches to this problem. One is the brute force approach to follow this program directly and to evaluate $I_\nu(s)$ along a large number of rays. This approach has been used in the SPUTTER program with one tangential ray through the center of each zone. Fig. 1-1 shows how these rays are combined to obtain the various values of $\bar{I}_\nu$ at the center of zone 4. Of the 7 rays which are drawn 3 are redundant because of symmetry and one obtains 3 different values of $\bar{I}_\nu$ going out, 3 going in and 1 grazing the zone.

In the other approach one defines certain moments, i.e. angular integrals of $\bar{I}_\nu$ which now depend only on the radius and not on the direction. To solve for these moments one integrates a system of coupled linear differential equations which are only approximately correct but which give exactly the right answer when one considers the limit where the diffusion approximation applies. Such schemes have been used widely in astrophysics and are discussed in great detail by Chandrasekhar (1960) and Mustel (1958). Some of the more sophisticated schemes use a large number of moments but quite good results can be obtained by restricting that number to two and using only the outgoing and ingoing flux which

are defined as the integrals

$$F_\nu\,(\overset{+}{-}) = \int \bar{I}_\nu \cos\theta \, d\Omega \qquad (1.4\text{-}4)$$

$$\cos\theta > 0$$
$$(\cos\theta < 0)$$

where θ is the angle between the ray and the radial direction.

We consider first the case of plane geometry where the medium is stratified in plane parallel layers. This geometry has been studied extensively by astrophysicists and applied to the radiative equilibrium in the outer regions of stars where it is indeed unnecessary to worry about the curvature. By treating the radial coordinate R as if it were a cartesian coordinate the angle θ of a ray remains constant along the ray path. The optical path length between two surfaces is therefore simply the optical path length along the normal divided by $|\cos\theta|$. We shall express this in terms of the optical depth conventionally defined by astrophysicists as the optical path length measured radially inward from the surface of a star (or in our case a fireball), i.e. the integral

$$\tilde{\tau}_\nu(R) = \int_R^{R_s} \tilde{\mu}_\nu(R')\, dR' \qquad (1.4\text{-}5)$$

To evaluate $I_\nu(s)$ as given by Eq. (1.4-2) for an outgoing ray we place s_o far enough inside that the factor $e^{-\tau_\nu}$ is essentially zero; for an ingoing ray we start at the surface where $\bar{I}_\nu(s_o) = o$. The first term of

Eq. (1.4-2) can therefore be left out in both cases. For the exponent under the integral one can write

$$\tau_\nu' - \tau_\nu = - \left| \frac{\tilde{\tau}_\nu' - \tilde{\tau}_\nu}{\cos \theta} \right| \qquad (1.4\text{-}6)$$

and for outgoing and ingoing rays one obtains

$$\bar{I}_\nu = \frac{1}{|\cos \theta|} \int_o^r \bar{B}_\nu(R') \, \bar{\mu}_\nu(R') \, e^{-\left| \frac{\tilde{\tau}_\nu' - \tilde{\tau}_\nu}{\cos \theta} \right|} dR' \; ; \; \cos \theta > o \qquad (1.4\text{-}7a)$$

$$\bar{I}_\nu = \frac{1}{|\cos \theta|} \int_r^{r_s} \bar{B}_\nu(R') \, \bar{\mu}_\nu(R') \, e^{-\left| \frac{\tilde{\tau}_\nu' - \tilde{\tau}_\nu}{\cos \theta} \right|} dR' \; ; \; \cos \theta < o \qquad (1.4\text{-}7b)$$

Entering these expressions into Eq. (1.4-4) one obtains the outgoing and the ingoing flux

$$F_{\nu+} = 2\pi \int_o^r \bar{B}_\nu(R') \, \bar{\mu}_\nu(R') \, E_2\left(|\tilde{\tau}_\nu' - \tilde{\tau}_\nu| \right) dR' \qquad (1.4\text{-}8a)$$

$$F_{\nu-} = 2\pi \int_r^{r_s} \bar{B}_\nu(R') \, \bar{\mu}_\nu(R') \, E_2\left(|\tilde{\tau}_\nu' - \tilde{\tau}_\nu| \right) dR' \qquad (1.4\text{-}8b)$$

where

$$E_2(\tau) = \int_1^\infty e^{-u\tau} \, u^{-2} \, du \qquad (1.4\text{-}9)$$

A useful approximation is obtained if one replaces $|\cos\theta|$ in Eq. (1.4-7) by an average $c = \overline{\cos\theta}$. Substituting the approximate form of $I_{\nu}^{\pm}$ into Eq. (1.4-4) gives expressions similar to those in Eqs. (1.4-8) but the exponential integral is now replaced by a simple exponential function, i.e. we have the approximation

$$E_2(\tau) \approx \frac{1}{2c}\ e^{-\tau/c} \tag{1.4-10}$$

The average intensities calculated from the approximate fluxes

$$\bar{I}_{\nu\,\pm} = F_{\nu\pm} / \pi \tag{1.4-11}$$

satisfy the differential equations

$$c\,\frac{d\bar{I}_{\nu\pm}}{dR} = \pm\,\bar{\mu}_{\nu}\,(\bar{B}_{\nu} - \bar{I}_{\nu\pm}) \tag{1.4-12}$$

These average intensities are therefore identical with the intensities in the directions for which $|\cos\theta| = c$. The idea of the two stream model with intensities in a characteristic direction goes back to Schwarzschild and Schuster who suggested to use $c = \frac{1}{2}$. A much better choice is $c = \frac{2}{3}$ which gives the correct net flux

$$F_{\nu+} - F_{\nu-} = -\frac{4\pi}{3\bar{\mu}_{\nu}}\,\frac{d\bar{B}_{\nu}}{dR} \tag{1.4-13}$$

in the high opacity limit.

In spherical geometry one has no simple rigorous expressions for I_ν like those given in Eqs. (1.4-7) from which to derive two stream equations. It seems, nevertheless, reasonable that one should be able to use equations which are essentially of the same character but with minor modifications to maintain conservation of energy. It is easy to see that this is achieved by the pair of equations

$$\frac{2}{3}\frac{d}{dR}\left(R^2 \bar{I}_{\nu\pm}\right) = \pm R^2 \bar{\mu}_\nu \left(\bar{B}_\nu - \bar{I}_{\nu\pm}\right) \tag{1.4-14}$$

With the definition given in Eq. (1.4-11) one obtains the outgoing and incoming flux simply by multiplying the corresponding intensities $\bar{I}_{\nu\pm}$ by a factor π . From the total integrated net flux

$$\mathcal{J} = \int (F_{\nu+} - F_{\nu-})\, d\nu \tag{1.4-15}$$

one can finally obtain the heating rate

$$\dot{Q} = -\frac{1}{R^2}\frac{d}{dR}\left(R^2 \mathcal{J}\right) \tag{1.4-16}$$

for use in the energy equation.

1.5 Finite difference equations

It is obviously impossible to find exact analytical solutions to the equations of RH and one must be satisfied with approximate numerical solutions. To obtain these one replaces infinitesimal increments of dependent as well as independent variables by finite differences. Mathematically RH

can be characterized as an initial value problem and the methods and problems arising in treating this by means of finite difference equations have been thoroughly discussed by Richtmyer (1957). We shall review some general considerations and then turn to questions which are specifically relevant to our problem.

In a finite difference scheme continuous variables are replaced by discrete ones but there are numerous possibilities for doing this. Thus one can regard the discrete values of a variable as representing either the values of the corresponding continuous variable at a set of discrete meshpoint or the average values between meshpoints. There are other variations but they are not needed in the following discussion. One can treat some variables in the first and others in the second manner. To indicate the actual choice one can use integral subscripts for variables defined at the meshpoints and half integral ones for those defined in the intervals. It is convenient and natural to let R_i and U_i represent the radius and the velocity of the particle at the meshpoint i and this leads almost automatically to defining $V_{i+1/2}$, $p_{i+1/2}$, $E_{i+1/2}$, and $T_{i+1/2}$ as the averages of specific volume, pressure, internal energy density and temperature in the interval between the meshpoints i and $i+1$.

Another element of choice enters in the methods used for advancing variables in time. Almost all variables are defined at meshpoints in time which are indicated by integral superscripts. It may, however, be useful to define the velocity between meshpoints which can be indicated by half integral superscripts. With this definition and abbreviating the right hand side of Eq. (1.2-3) by a (for acceleration) that equation and Eq. (1.2-2)

lead to the integration procedure

$$U^{n+1/2} = U^{n-1/2} + a^n \delta t \tag{1.5-1}$$

$$R^{n+1} = R^n + U^{n+1/2} \delta t \tag{1.5-2}$$

Having obtained R^{n+1} one can then obtain V^{n+1} by differencing which follows from Eq. (1.2-1). So far we have not bothered to look at alternate schemes because the procedures outlined above are very straightforward and there seems to be no good reason for doing anything more elaborate. In the purely hydrodynamic case, i.e. if $\dot{Q} = 0$ the energy equation (1.2-4) can also be integrated very simply. Centering the difference equation at (n+1/2) leads to

$$E^{n+1} - E^n + \frac{1}{2}\left(p^{n+1} + p^n + 2q^{n+1/2}\right)\left(V^{n+1} - V^n\right) = 0 \tag{1.5-3}$$

If E is expressed as a function of V and p this equation can be solved for p^{n+1}, the one variable which is still unknown. Anticipating the problems which arise when one has radiative heating it is really more useful to express both E and p as functions of V and T and to solve Eq. (1.5-3) for T^{n+1}. Either way one has to solve for only one unknown at a time which causes no real difficulty even though it may have to be done by iteration.

This situation changes drastically when the radiative heating rate $\dot{Q}$ becomes important. The heating term to be added on the right hand side of Eq. (1.5-3) should be centered at the level $t^{n+1/2}$ like the remaining

part of the equation but the temperature distribution from which it must be calculated is only known up to the time t^n . There are two major avenues of attack. One is to forget about centering $\dot{Q}$ and use its value as calculated at t^n . If this is done one can still solve explicitly for T^{n+1} and this is as the explicit method of integration.

In the other attack one uses the properly centered heating rate $\frac{1}{2}\left(\dot{Q}^{n+1} + \dot{Q}^n\right)$. This means that the equation which describes the heating in any one zone depends on the values of T^{n+1} in all zones so that one has to solve a large number of equations (one per zone) simultaneously. This implicit method involves a considerable amount of algebraic labor. If centering was only required for accuracy it would not be worthwhile to go to all this trouble because one could increase the accuracy more easily by reducing δt . What is really involved is the question of mathematical stability which we shall briefly discuss.

It is physically clear that a fluid responds to any pressure or temperature disturbance by a motion or heat flow which counteracts the disturbance. In an integration by means of difference equations which uses too large time intervals it may happen that the disturbance is overcompensated so that an excess turns in one step into a deficit, in the next step again into an excess etc. If the magnitude of this alternating disturbance increases each time any small disturbance will eventually cause the solution to blow up. In principle one can cure such an instability by taking δt small enough but this could seriously increase the running time of a problem.

There are two cases where the stability condition has been obtained analytically. The first arises when the dominant mode of energy transfer is of a hydrodynamic nature. The maximum δt in this case is found as

follows. One calculates for each zone the traversal time $\Delta t_{i+1/2} = \left(R_{i+1}-R_i\right)/V_{s(i+1/2)}$ of a signal traveling with the local sound speed. Going through all intervals one then finds the smallest, say Δt_{min}. The time increment is then limited by the so-called Courant-condition

$$\delta t < k \Delta t_{min} \tag{1.5-4}$$

where k is a numerical factor near unity which depends on the integration scheme. In the scheme where one uses the three equations at the beginning of this section one has $k = 1$.

When radiative heating dominates, the stability analysis has been carried out for the case where one can use the diffusion approximation. In the explicit scheme the limit for δt is proportional to $\bar{\mu}_R \delta R^2$ which decreases together with the Rosseland mean absorption coefficient of the air. If the air is fairly transparent δt is limited to very small values and this makes an explicit calculation very costly in computer time. The implicit method does not have this trouble and is in fact unconditionally stable. On the other hand it is of course also time consuming to solve a large number of coupled equations simultaneously. One can attempt to approach the implicit solution by iteration. On the first go-around one can advance T by the explicit method. With the advanced temperature distribution one can then work out $\dot{Q}^{n+1}$, form the average $\dot{Q}^{n+1/2} = \frac{1}{2}\left(\dot{Q}^n + \dot{Q}^{n+1}\right)$ and reevaluate T^{n+1}. This procedure can be repeated several times and if it converges it will lead to a stable solution. The time step δt is now limited by the condition that the solution should converge. In contrast to the stability condition of the explicit method this limit of δt

is inversely proportional to $\bar{\mu}_R$ and independent of the zone size. The actual convergence criterion is almost equivalent with imposing a limit on the fractional energy change per time step within every zone. That form of the condition is easy to use and experience has shown that a fraction like one percent ensures the convergence. In a purely implicit procedure there is no such limitation on the magnitude of the time step. For the sake of accuracy one should also impose a limit on the fractional energy change per time step but it does not need to be as small. This limit can be allowed to vary from zone to zone to require greater accuracy in those zones where the changes make a significant contribution to the overall picture.

It is obvious that the allowed time interval changes throughout the calculation. For reasons of economy one should always run fairly close to the maximum without, however increasing δt too abruptly. To change δt generally requires some interpolation (or extrapolation) and all programs nowadays have provisions for carrying the necessary changes out automatically. Although the preceding arguments were based on the diffusion approximation they apply equally in the more general case. It is true that one can not readily obtain analytic stability or convergence criteria but experience with numerical calculations indicates the same pattern.

In addition to the various decisions described above one also has to make a choice on zone sizes. There are two parts to this decision relating to the total number of zones and to their relative sizes at different radii. Part one involves a compromise between conflicting requirements for accuracy and economy because it takes a large amount of computer time to use very many zones. This is amplified if the choice of δR also limits the time step as in hydrodynamic calculations where $\delta t \sim \delta R$ and even more in the explicit

calculation of radiative transfer where $\delta t \sim \delta R^2$.

Part two involves a judgment as to where the really significant changes are taking place and it is of course at those regions where one should use the finest zoning. In the course of a calculation the location of significant changes moves so that one has to make provisions in the program to detect this and to react to it by rezoning. Furthermore the overall radius of the fireball changes during an average calculation by as much as 3 orders of magnitude so that rezoning is also necessary to keep the number of zones at a more or less constant level.

The pseudo viscous pressure q introduced in Eq. (1.2-5) is a device for calculating the entropy rise behind the shock. Without the damping mechanism provided by q the changes induced behind the shock overshoot and produce lasting oscillations which physically do not belong there. A large value of ℓ will kill these improper oscillations most effectively but at the cost of making the transition region very wide which is also incorrect. Experience has shown that $\ell = 2\Delta R$ will stop the fake oscillations reasonably fast without spreading the shock transition over more than about 4 zones.

One can also express q in terms of $\frac{\partial V}{\partial t}$ rather than $\frac{\partial u}{\partial r}$ and Richtmyer suggests to use the formula

$$q = \frac{(\rho_o \ell)^2}{V} \left(\frac{\partial V}{\partial t}\right)^2 , \quad \frac{\partial V}{\partial t} < 0 \qquad (1.5\text{-}5)$$

with

$$\ell = a \left(\frac{r}{R}\right)^2 \Delta r \qquad (1.5\text{-}6)$$

so that the transition region covers the same number of zones near the center and further away from it. The numerical factor a should again be approximately 2.

In differencing either Eq. (1.2-5) or (1.5-5) one is led to expressions at half integral times. To obtain the acceleration in Eq. (1.5-1) it should be known at t^n but to achieve that, one would have to use an implicit routine. In this case that is not worthwhile since the use of q is an artifice anyway and it is customary to have q lag half a time step behind. In the energy equation (1.5-3) q is automatically in step.

The total energy which is obtained by summing the kinetic and internal energy within the fireball and the energy carried away by radiation should always stay at a constant level. The internal energy should in principle contain a part due to radiation but as mentioned in section 1.1 this does not amount to much. A trivial point, but one which must nevertheless be kept in mind is, that one should only count the excess over the energy in the ambient unheated air; otherwise the nominal energy would grow with the volume of the fireball.

It is important to keep track of any violations of energy conservation which may creep in through the use of finite difference schemes. Any program should therefore contains a routine for checking energy conservation.

The point at which R.H. goes beyond standard methods comes with the calculation of radiative transfer. The various methods require the evaluation of certain space integrals before one can calculate the energy deposition in a specified zone. Because of the very strong temperature dependence of the integrands these integrals depend critically on the radial dependence of the temperature. The common method of approximating

this dependence by assuming constant values of the temperature within the zones may lead to serious errors. An attempt to correct these has been made by Hillendahl (1964).

We shall present the analysis for the plane case which is formally easier. The transition to spherical geometry can be made later and requires only minor changes which are rather obvious. The starting point is Eq. (1.4-12) but before one has carried out any frequency averaging. Thus the line character is still preserved and μ_ν and $I_{\nu\pm}$ are rapidly changing functions of frequency. Integrating Eq. (1.4-12) across the zones which are separated by the interface at R_i one finds for the outgoing and ingoing stream (represented by the upper and lower sign

$$I_{\nu\pm,i} = I_{\nu\pm,i\mp1}\, e^{-\frac{3}{2}\Delta_{\nu,i\mp\frac{1}{2}}} \pm \frac{3}{2}\int_{\tilde\tau_{\nu,i}}^{\tilde\tau_{\nu,i\mp1}} B'_\nu\, e^{-\frac{3}{2}\Delta'_{\nu,i}}\, d\tilde\tau'_\nu \qquad (1.5\text{-}7)$$

where B'_ν and $\tilde\tau'_\nu$ are taken at the same point R', and

$$\Delta'_{\nu,i} = |\tilde\tau_{\nu,i} - \tilde\tau'_\nu| \quad ; \quad \Delta_{\nu,i+1/2} = |\tilde\tau_{\nu,i} - \tilde\tau_{\nu,i+1}| \qquad (1.5\text{-}8)$$

To carry out the integral we use the first two terms of the power expansion

$$B'_\nu = B_{\nu,i} + \left(\frac{dB_\nu}{d\tilde\tau_\nu}\right)_i \left(\tilde\tau'_\nu - \tilde\tau_{\nu,i}\right) + \ldots \qquad (1.5\text{-}9)$$

and obtain

$$I_{\nu\pm,i} = I_{\nu\pm,i\mp1}\, e^{-\frac{3}{2}\Delta_{\nu,i\mp1/2}} + B_{\nu,i}\left(1 - e^{-\frac{3}{2}\Delta_{\nu,i\mp1/2}}\right)$$

$$\pm\frac{2}{3}\left(\frac{dB_\nu}{d\tilde{\tau}_\nu}\right)\left[1 - \left(1 + \frac{3}{2}\Delta_{\nu,i\mp1/2}\right)e^{-\frac{3}{2}\Delta_{\nu,i\mp1/2}}\right] \qquad (1.5\text{-}10)$$

Integrating over frequency we obtain formally

$$I_{\pm,i} = I_{\pm,i\mp1}\; Z_{\pm,i} + \left(B_i\; 1 - A_{\pm,i}\right) \pm \frac{2}{3}\left(\frac{dB}{d\tilde{\tau}}\right)_i\; W_{\pm,i} \qquad (1.5\text{-}11)$$

where

$$I_{\pm,i} = \int I_{\nu\pm,i}\, d\nu \qquad (1.5\text{-}12)$$

$$Z_{\pm,i} = \int \frac{I_{\nu\pm,i}}{I_{\pm,i}}\; e^{-\frac{3}{2}\Delta,i\mp1/2}\; d\nu \qquad (1.5\text{-}13)$$

$$A_{\pm,i} = \int \frac{B_{\nu,i}}{B_i}\; e^{-\frac{3}{2}\Delta_{\nu,i\mp1/2}}\; d\nu \qquad (1.5\text{-}14)$$

$$W_{\pm,i} = \int \frac{\left(\frac{dB_\nu}{d\tilde{\tau}_\nu}\right)}{\left(\frac{dB}{d\tau}\right)_i}\left[1 - \left(1 + \frac{3}{2}\Delta_{\nu,i\mp1/2}\right)e^{-\frac{3}{2}\Delta_{\nu,i\mp1/2}}\right] \qquad (1.5\text{-}15)$$

and where B_i is the integrated intensity

$$B = \int B_\nu\, d\nu = \frac{\sigma}{\pi}\, T^4 \qquad (1.5\text{-}16)$$

at the point R_i . No limits of integration have so far been specified and one is free to divide the spectrum into any set of frequency intervals. As a first approximation Hillendahl used the entire spectrum without subdividing it.

Before Eq. (1.5-11) can become operational one has to define the average optical depth $\tilde{\tau}$ which enters in the derivative $\frac{dB}{d\tilde{\tau}}$ and one has to face the difficulty of an unknown ratio I_ν/I entering into the definition of Z .

The procedure devised by Hillendahl for obtaining an average for $\tilde{\tau}$ is specifically intended for use with finite zone sizes. The prescription is designed to keep the emissivity of a zone of constant density and temperature unchanged if one replaces the frequency dependent optical depth $\Delta\tilde{\tau}_\nu = \mu_\nu \Delta R$ by its average $\Delta\tilde{\tau}$. Thus, i.e., by making this substitution in the exponent of Eq. (1.5-14) one is led to

$$A = \int \frac{B_\nu}{B} e^{-\frac{3}{2}\mu_\nu \Delta R} d\nu = e^{-\frac{3}{2}\Delta\tilde{\tau}} \qquad (1.5\text{-}17)$$

We note that the factor $\frac{B_{\nu,i}}{B_i}$ in Eq. (1.5-14) is taken at the edge of the zone. Since this ratio varies only very slowly with T we will take it at the center of the zone instead, so that there is only one emissivity A per zone and not different ones for the ingoing and outgoing ray. In the above integral for A one can clearly replace the rapidly varying exponential by a smooth one in which one uses the slab absorption coefficient $\bar{\mu}_\nu$ as defined in Eqs. (1.3-2) and (1.3-3). In

the new expression

$$A = \int \frac{B_\nu}{B} e^{-\frac{3}{2}\bar{\mu}_\nu \Delta R} d\nu \qquad (1.5\text{-}18)$$

the zone with ΔR enters not only as a factor in the exponent but also as one of the variables in $\bar{\mu}_\nu = \bar{\mu}_\nu(\rho, T, \Delta R)$. Various methods for calculating $\bar{\mu}_\nu$, which apply when the dominant absorption is molecular, atomic or free-free, have been described in (2). The results are tabulated in and have been used in the above integral to obtain $A(\rho, T, \Delta R)$. It is convenient to express this in terms of a mean absorption coefficient $\bar{\mu}_H(\rho, T, \Delta R) = -\frac{2}{3} \ell n\, A/\Delta R$ and to write:

$$A = e^{-\frac{3}{2}(\bar{\mu}_H \Delta R)} \qquad (1.5\text{-}19)$$

Depending on ΔR as well as on ρ and T, $\bar{\mu}_H$ differs from the Rosseland mean $(\bar{\mu}_R)$ and the Planck mean $(\bar{\mu}_P)$ which depend only on ρ and T .

For the function W , which should in principle be calculated from Eq. (1.5-15) we use an approximation and set

$$W = 1 - \left(1 + \frac{3}{2}\bar{\mu}_H \Delta R\right) e^{-\frac{3}{2}\bar{\mu}_H \Delta R} \qquad (1.5\text{-}20)$$

which looks reasonable and leads to the correct energy deposition when $\bar{\mu}_H \Delta R$ is large enough that one can use the diffusion approximation. As in the case of A we are using only one W per zone.

The coefficients $Z_{\pm,i}$ as defined by Eq. (1.5-13) depend on the unknown spectral distribution $I_{\nu\pm,i}/I_{\pm,i}$ at the point i. Since the difference equations (1.5-11) apply only to the integrated intensities $I_{\pm,i}$ their solution does not give us any direct information about the spectral distribution and we must rely on educated guesses for the latter. The basic clue which we follow is that the radiation at some point i comes by and large from a zone (the radiating zone) which lies an optical depth unity behind that point in the direction where the stream comes from. The distribution has therefore the distribution of a blackbody source at the temperature T_R of the radiating zone but modified by selective absorption in the intermediate zones.

When we apply this model we distinguish 3 typical situations for the ingoing and 3 for the outgoing stream. This comes from the peculiar temperature dependence of mean absorption coefficients. For all these means, whether we talk of $\bar{\mu}_R$, $\bar{\mu}_P$ or $\bar{\mu}_H$, one can distinguish a central temperature range where $\bar{\mu}$ is large and the low and high T ranges where it drops to very low values.

The temperature profile of a fireball is typically a monotonically decreasing curve. While the central temperature is still large this profile looks like the sketch in Fig. 1-2 with a central section where $\bar{\mu}$ is small so that radiative transfer maintains a nearly constant temperature. Beyond that plateau comes a more opaque region with a relatively large temperature gradient and at the point where the temperature has dropped to where the air is again transparent the profile becomes again more level. Superimposed on this one finds usually

some structure due to shocks or other disturbances but this does not alter the main conclusion that there are 3 distinct regions.

In the opaque intermediate region the spectral distribution I_ν/I can be identified with that of a blackbody at the local temperature so that one is led to $Z_+ = Z_- = A$. In the interior region the radiation comes mainly from its boundary where it is in contact with the opaque region. Since the temperature profile is quite level one does not commit a significant error by identifying I_ν/I again with the local B_ν/B which varies much less with T than either B_ν or B itself. As in the previous case we therefore use the approximation $Z_+ = Z_- = A$.

Only in the outer section do we have to make a more careful choice of I_ν/I and only for the outgoing stream. The ingoing stream carries essentially no energy and it doesn't matter much what one does. The simplest choice is again to set $Z_- = A$.

The point where it really counts that our model should adequately represent the true physical nature of radiative transport comes when we consider the outgoing stream as it emerges from the opaque region. The absorption to which this stream is subjected is largely due to molecules. In calculating which parts of the spectrum are and which are not transmitted one is greatly helped by the character of the energy dependence of $\bar{\mu}_\nu$. Fig. 1-3, which is a typical example taken from SACHA type calculations shows that $\bar{\mu}_\nu$ is a very rapidly rising function of frequency. From this graph we find by inspection that a zone of about 10 m thickness would transmit practically no photons above 5 eV and practically all photons below 3.8 eV. Approximately one can assume that there is a critical photon energy $h\nu_c$ in the

vicinity of 4.4 eV at which the transmitted flux is sharply cut off. For a stream which starts out as a blackbody spectrum $\frac{B_\nu}{B} = b_\nu (T_R)$ one finds that the transmitted fraction of the energy is a known integral

$$L (T_R, h\nu_c) = \int_0^{\nu_c} b_\nu (T_R)\, d\nu \qquad (1.5\text{-}21)$$

The procedure for determining Z_+ for any zone $i - 1/2$ outside the opaque region starts out with finding the radiating zone which belongs to it and whose temperature has been designated as T_R. We assume that the model spectrum I_ν starts out there as a blackbody spectrum $B_\nu (T_R)$. Any of the zones through which it passes will not transmit any radiation above its cut-off energy $h\nu_c$ and the model spectrum which finally enters into zone $i - 1/2$ remains $I_\nu = B_\nu (T_R)$ up to the lowest cut-off energy $h\nu_{min}$ encountered by the stream but is reduced to $I_\nu = 0$ above $h\nu_{min}$. If zone $i - 1/2$ has a lower cut-off $h\nu_{c,\, i-1/2}$ we set therefore

$$Z_{+,i} = \frac{L\left(T_R\, h\nu_{c,\, i-1/2}\right)}{L\left(T_R\, h\nu_{min}\right)} \qquad (1.5\text{-}22)$$

If the cut-off energy is equal to or larger than $h\nu_{min}$ the model spectrum would lead to $Z_{+,i} = 1$ which can't be right and indicates that the sharp cut-off approximation is too crude to fit this case. An upper limit for Z can be obtained by setting it equal to the local A since the bulk of the true spectral distribution lies at somewhat higher energies than the local blackbody spectrum. Thus the true I_ν will suffer somewhat more absorption than B_ν which leads to the inequality

$Z < A$.

The modifications which are necessitated by going to spherical symmetry can be written down without difficulty. One only has to note that Eq. (1.4-14) differs from Eq. (1.4-12) by the factor R^2 which multiplies both I and B. Clearly this factor must enter when one modifies the corresponding set of Eqs. (1.5-11). As we write down the modified set we incorporate the result that the coefficients A and W depend only on the zone and not on the direction of the stream, and obtain:

$$R_i^2\, I_{\pm,i} = R_{i\mp 1}^2\, Z_{\pm,i}\, I_{\pm,i\mp 1} + R_i^2\, B_i \left(1 - A_{i\mp 1/2}\right) \pm \frac{2}{3} R_i^2 \left(\frac{dB}{d\tilde{\tau}}\right)_i W_{i\mp 1/2} \qquad (1.5\text{-}23)$$

The A and W are as before given by Eqs. (1.5-19) and (1.5-20) and $Z_{\pm,i}$ is nearly always equal to $A_{i\mp 1/2}$ except in a few zones just outside the opaque region where one should use Eq. (1.5-22).

In the two stream model the fluxes differ from the intensities by a factor π as shown in Eq. (1.4-11). The relation carries over when one performs the frequency radiation so that

$$F_{\pm,i} = \pi\, I_{\pm,i} \qquad (1.5\text{-}24)$$

Having determined the intensities by solving the set of Eqs. (1.5-23) one is therefore ready to evaluate $-\nabla \cdot \vec{F}$ which according to Eq. (1.2-8) gives us the radiative heating rate. Thus one obtains:

$$\dot{Q}_{i+1/2} = \frac{3}{R_{i+1}^3 - R_i^3} \left[R_i^2 \left(F_{+,i} - F_{-,i}\right) - R_{i+1}^2 \left(F_{+,i+1} - F_{-,i+1}\right) \right] \qquad (1.5\text{-}25)$$

which is the final equation of the two stream method. Before leaving it let us take a somewhat closer look how this equation handles a zone in the opaque region. In any region where one replaces Z_+ and Z_- by A one finds that Eq. (1.5-23) reduces to:

$$I_{\pm,i} = B_i \pm \frac{2}{3}\left(\frac{dB}{d\tilde{\tau}}\right)_i \quad W_{i\mp 1/2} \tag{1.5-26}$$

from which one obtains in turn

$$F_{+,i} - F_{-,i} = \frac{2\pi}{3}\left(\frac{dB}{d\tilde{\tau}}\right)_i \left(W_{i-1/2} + W_{i+1/2}\right) \tag{1.5-27}$$

In an opaque region this simplifies still further because the factor $e^{-\frac{3}{2}\bar{\mu}_H \Delta R}$ in Eq. (1.5-20) becomes negligible compared to unity. One can therefore set $W = 1$ and obtain:

$$F_{+,i} - F_{-,i} = \frac{4\pi}{3}\left(\frac{dB}{d\tilde{\tau}}\right)_i \tag{1.5-28}$$

and since $B = \frac{\sigma}{\pi} T^4$ this is clearly equivalent to Eq. (1.2-9) i.e., to the basic equation of the diffusion approximation. This is of course no surprise because we picked $\overline{\cos\theta} = \frac{2}{3}$ precisely in order to achieve this equivalence.

In an opaque region, the diffusion approximations contains all the physics needed for the calculation of radiative energy transfer and it is superior to other methods as far as speed and possibly accuracy of calculation are concerned. Hillendahl's formulation of the two stream

method automatically leads to this procedure. More elaborate methods such as the multiple ray technique do not, but it is of course possible to switch to a diffusion theory calculation when one considers an opaque region. This is indeed done in the SPUTTER program.

In the form outlined in this section the two stream method is not applicable at high altitudes where the ambient air has a density less than about $\rho/\rho_o = 10^{-4}$. At such densities the air becomes transparent in the spectral region where B_ν has its maximum and the fireball has no opaque region. There is still a significant amount of radiation at frequencies above and below this window but one has to devise new methods for dealing with this problem. At these altitudes the mathematical difficulties are further aggravated by the non-spherical energy deposition which takes place when the mean free path of x-rays get large compared to the atmospheric scale height. Eventually, say at about $\rho/\rho_o \approx 3 \times 10^{-6}$ the air becomes transparent in all parts of the spectrum and thermal radiation is no longer a significant factor.

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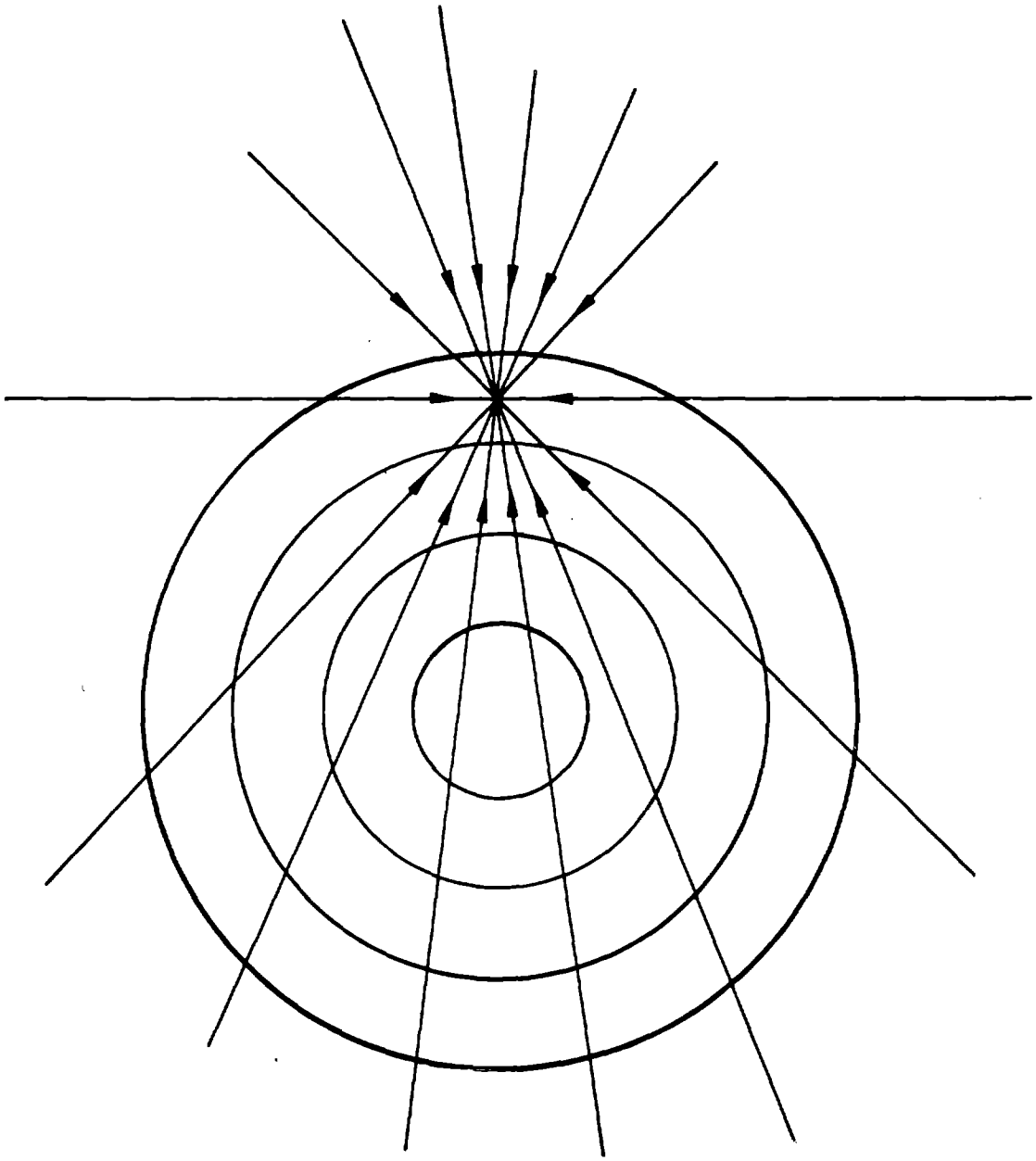

FIG. 1-1 RAYS CONVERGING UPON ZONE 4 WHICH ARE USED TO COMPUTE I_ν AS FUNCTION OF ANGLE

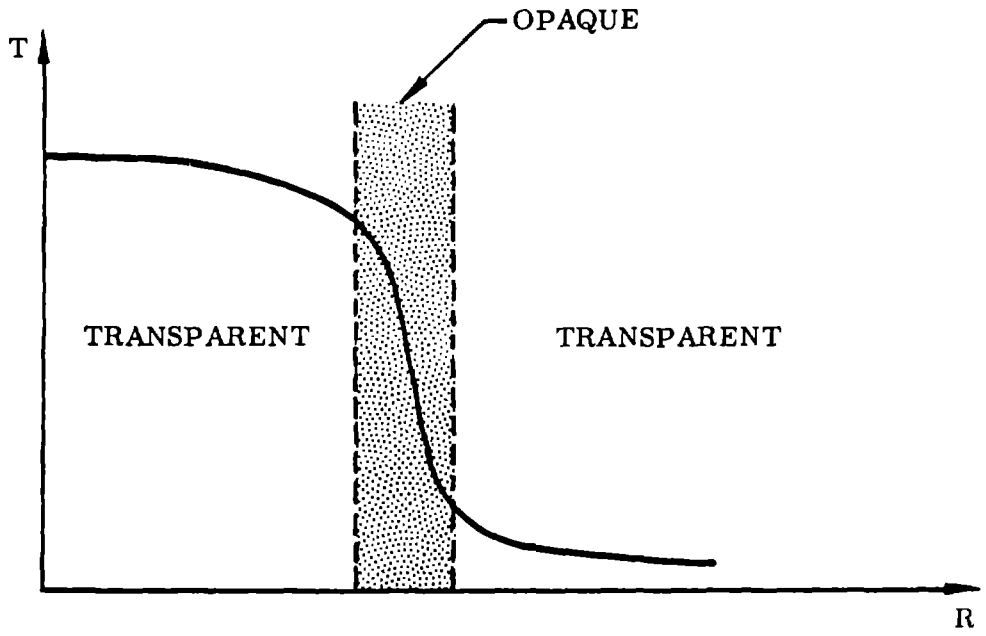

FIG. 1-2 SKETCH OF TEMPERATURE PROFILE IN A FIREBALL INDICATING OPTICAL PROPERTY OF THE AIR

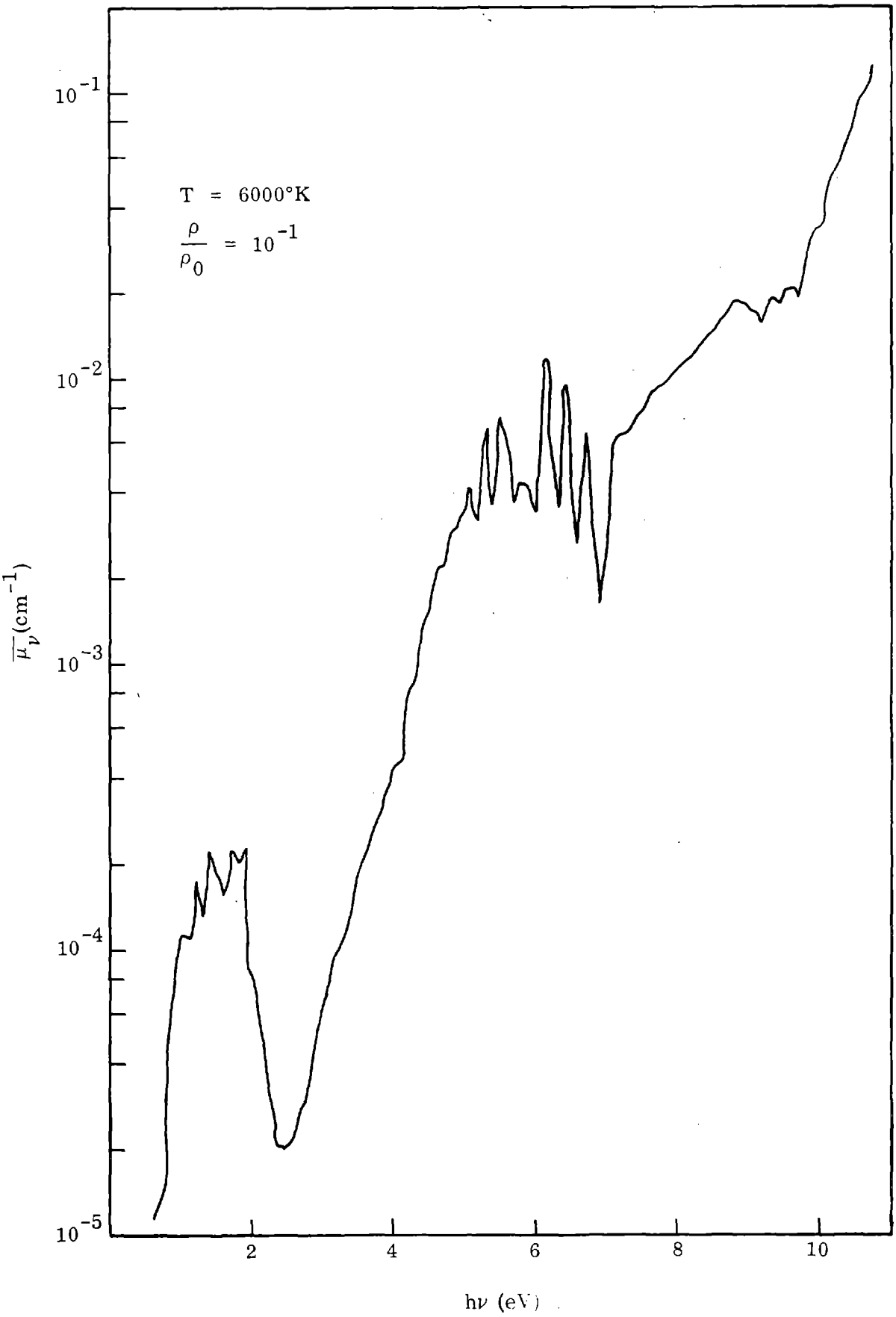

FIG. 1-3 TYPICAL ENERGY DEPENDENCE OF ABSORPTION COEFFICIENT

Chapter 2. THE PHYSICS OF FIREBALLS

2.1 Introduction

A nuclear explosion in the atmosphere creates a fireball whose development depends in large measure on the physics of hot air. All of the previously discussed properties of hot air and all of the mechanisms for energy transport developed in previous chapters are a part of nuclear fireball physics. However, these energy transforming and transporting relations and the detailed knowledge of the properties of air find considerably wider application. They have or can contribute to the study of stellar dynamics, the nature of stellar atmospheres, the radiation from various astrophysical sources, and they can aid in the study of hypervelocity flight, upper atmosphere physics, aurora, and other atomic and molecular physics problems which involve high temperatures.

It is certainly the case that the information presented in these previous chapters makes the conditions created in a nuclear explosion more understandable. Some knowledge of air heating mechanisms, of air excitation, of radiation transport, and of hydrodynamics, of absorption properties, and of the thermodynamics of air is necessary before a full description of a nuclear explosion can become more than heuristic.

Much of the present knowledge about fireballs has been gleaned from test observations, but by far the greatest detail has come from numerical computer calculations, as have the quantitative estimates of fireball interior dynamics which appear in this chapter. Calculations of widely varying detail and sophistication now abound, and it is not the intention in this chapter to review such results or analyze computing methods. Most current calculations rely for their measure of success on the extent to which the physical concepts and properties covered in the

preceding chapters have been taken into account in a mathematical model. The principal objective of this chapter is to outline the physical features of nuclear fireballs and their thermal radiations, stressing where possible those factors which are most general and which provide the best understanding on which to base predictions and extrapolations. The approach adopted is to begin by considering a small yield explosion (1 kiloton) at sea level and to describe the sequence of events which occur unencumbered with interactions from the earth's surface or inhomogeneous environments. This development will then be extended to higher yields and altitudes. There will be no attempt at completeness and no great concern for quantitative rigor, but it is intended to display as much as possible the current understanding of the physics of nuclear fireballs.

2.2 One kiloton at sea level

A one kiloton explosion in a sea level atmosphere provides an appropriate example for an initial examination of the sequence of events that constitute a fireball history. The now familiar usage of kilotonage and megatonage refers to the total energy release in a nuclear explosion with the usual metric prefixes for a thousand or a million and with the understanding that a ton of high explosive - TNT - releases 10^9 calories of effective energy, i.e., one gram of TNT is taken as equivalent to one kilocalorie or 4.185×10^{10} ergs.

For any nuclear explosion the sequence of events is remarkably complex. In following its development for this one kiloton sea level explosion, the reader may appreciate that the present understanding,

although not complete, has become quite detailed and much of it has grown directly from the material reviewed in this series of volumes.

The nuclear energy is released in an extremely short time - a small fraction of a microsecond - and always in a small mass and volume. It is the properties of this small mass, constituting the weapon itself and its carrier that determine the early source of energy for the fireball, and some of these properties may influence the character of the later thermal radiation. Everything starts in this nuclear source and all of the initial radiations - gamma rays, neutrons, and x-rays – are generated by it. However, the air or other immediately surrounding material absorbs almost everything emitted within a few hundred meters and the nature of the observable fireball is largely determined by the properties of this surrounding air. For our example of one kiloton in a sea level atmosphere, the air within a few feet of the weapon stops nearly all of the x-rays, and the prompt gamma rays and the neutrons have removal mean free paths of about 400 and 240 meters, respectively. These rapid absorptions make knowledge of specific details of the nuclear device largely unnecessary in describing the fireball phenomena. Consequently, we shall be able to proceed without reference to classified aspects of nuclear weapons and yet without significantly truncating our description of the fireball and its thermal radiations.

The fraction of the energy which may be radiated out of the weapon as x-rays before it begins to blow apart under hydrodynamic action depends largely on its yield-to-mass ratio and to some extent on other construction details. This fraction may range from almost nothing at all (or a very small percent) to a significantly more than 80% of the total

energy generated (Glasstone, 1962; Brode, 1964 b).

Before the air has had a chance to re-radiate any of the energy deposited by the x-rays, the bulk of this energy is concentrated in a relatively small sphere and at a temperature which is typically of the order of several million degrees K. There is, however, a small fraction of x-rays, from the high frequency end of the spectral distribution function, which penetrates to a distance of perhaps a few meters, and heats this shell to temperatures in the 10,000°K range. Energywise this heating is insignificant but it makes a contribution to the fireball phenomenology which is of some interest. By consulting the table of mean free paths on p. 447 of (3)* we discover that this shell is opaque. As long as it exists such an opaque shell hides the much hotter sphere on its inside and all that can be observed is the radiation from the shell itself, which is comparatively dim.

This phase is always very short-lived and terminates when the radiation from the center floods into the shell and heats it up. During the next phase the fireball can be characterized rather well as an extremely high temperature sphere of air surrounding the nuclear source and showing a fairly sharp temperature drop at its edge. The interior of this high temperature sphere may be at a fairly uniform temperature, and the whole may contain quite a large fraction of the nuclear explosion energy in the form of heat. Some small fraction always remains in the dense bomb vapors, but most of the early phases of the fireball development are quite independent of the details of the weapon design. The subsequent explosion and radiation behavior can be derived almost entirely from the properties of this hot air. Such a model will be less true in high altitude or space

* DASA-1917-3.

environments where the immediate external surroundings fail to contain as thoroughly the explosion energy because they lack sufficient opacity or optical thickness.

Throughout the explosion development, radiant energy is emitted by the fireball. That fraction which is transmitted by the cold exterior atmosphere is called thermal radiation. The rate of energy emission, or radiated power, has shape as shown in Fig. 2-1. If the opaque fringe layer has been penetrated early enough and if the instruments used for measuring the thermal power have sufficient time resolutions, the signal will include the early peak which is shown as a dotted line. Otherwise, one sees the two-peak curve which is drawn as a full line. The explanation of this curve is an important objective of any theory.

Although it is a rather simple exercise, it is instructive to note the rather small size of air volumes required to contain the large amounts of energy at the high temperatures created by the absorption of the initial flux of x-rays. The following table indicates the radii of spheres for a few examples of energy content and temperatures. These temperatures, of course, are too high for the air to remain that hot for very long, but in the immediate first fractions of a microsecond these radii are representative of the sizes and temperatures of the earliest (x-ray) fireballs.

Size of Spheres of Sea Level Air Necessary to Contain 1 KT, 1 MT or 100 MT of Energy at Various Uniform Temperatures

Temperature Millions of ^{o}K	1 KT	1 MT	100 MT
7 1/2	3/4 m	7.5 m	35 m
6	1	10	46
5	1 1/4	12	57
4	1.6	16	74
3	2.1	21	100

In a very few microseconds, these fireballs would have grown much larger and much less hot by the continued diffusion of radiation into the external cold air.

For most considerations these earliest phases of x-ray deposition and re-radiation remain both obscure and of little probable importance. When the flux of source radiations has been sufficiently intense as to completely strip the electrons from the air ions, then that volume of plasma can offer only Compton scattering as further resistance to the x-ray flux or as opacity to its own re-radiation. The most appropriate physical model for the continued expansion of this low emissivity, high energy density region is neither by hydrodynamics (which requires relatively long times to accelerate masses of gas) nor by radiation diffusion which presumes many interactions over any appreciable temperature gradient. The growth of such a heated volume is a radiative process which can be characterized roughly by its emissivity, temperature, and volume together with the heat capacity of the external cold air. The single further physical characteristic necessary to include in a growth rate prediction is the fact that the surrounding air is essentially opaque to the radiations from this hot air. Detailed knowledge of the opacity between this blackness at cold temperatures and its transparent nature at sufficiently high temperature is at this point unnecessary. Thus, the rate of energy lost, expressed as a grey-body loss rate, is the rate at which energy is deposited in the cold air at the surface of the high temperature sphere, viz.,

$$\frac{dW}{dt} = 4\pi R^2 \sigma T^4 e \quad , \qquad (2.2\text{-}1)$$

in which e is the emissivity, T the temperature of the hot isothermal sphere, R its radius, and dW/dt the rate of energy change. When an appropriate specific heat is introduced, a differential prescription for the volume growth and temperature drop results.

Following this approximation, one can express the rate of growth, dR/dt, in the same terms as

$$\frac{dR}{dt} = \frac{e\sigma T^4}{E\rho} \quad , \tag{2.2-2}$$

in which E represents the internal energy per unit mass, and ρ the density of the air just behind the front, while T is the inner temperature. The usefulness of this approximation in estimating the rate of growth of a partially transparent fireball is largely dependent on the accuracy with which average or "effective" interior temperatures, specific energies, and emissivities can be chosen. During the most rapid expansion, the interior is likely to be considerably non-isothermal, i.e., the interior may be more than twice as hot as the region just behind the front. The dependence on the fourth power of the temperature makes this rate quite sensitive to such differences. The most uncertain quantity is likely to be the effective emissivity, since it represents some average over the emitting region, and may also disguise some geometric dependence - not all the radiation being emitted radially. Appropriate choices of effective emissivity and temperature may make this simple formula appropriate for predicting the growth rate during the subsequent radiation diffusion phase.

The temperature profiles illustrated in Fig. 2-2 are typical of this early radiative growth for a one kiloton sea level burst. The curves represent the air temperatures as a function of radius for six selected instants in time. The dashed curve indicates the shock temperatures. It is the lowest temperature within the fireball at each of these times. After about 15 microseconds, the radiation diffusion growth becomes so slow that a shock wave begins to form, to compress the newly engulfed air and heat it to a temperature substantially below that of the radiatively heated inner sphere. With either the early radiative expansion or the subsequent adiabatic expansion behind the forming shock front, the inner temperatures drop with time in an approximately exponential fashion. During this early growth, the power radiated or the thermal radiation to points outside the fireball is not a significant fraction of the energy it contains. The time is short, the size is small, its opacities are high, and the fireball exterior so well shields the hotter core that the radiation out is less than half a per cent of the available energy.

Of course, the radiative properties are influenced by the air density as well as by the temperature, and the gradual formation of the shock causes an appreciable increase in the air density at the fireball surface (as much as tenfold increase at sea level). In the process, the outer surface of the fireball passes from a rather diffuse radiation-driven front to a sharp, dense shock front. Fig. 2-3 shows some typical early density profiles, in which the shock is seen to grow and the fireball interior is seen to expand to much less than the external ambient density.

Reference to the opacities for air as given in Volume 3 will

confirm that the shock front at these early densities and temperatures is quite opaque. For instance, at the 250 μsec time of Figs. 2-2 and 2-3, the emission mean free path for a shock temperature of 30,000°K and a density of 8 times normal is about 0.01 cm. The fireball will expand to much lower temperatures and much larger size before anything behind the shock front will become visible. It is during this period that the thermal radiation rate decreases toward a minimum and the fireball appears to grow dimmer. (Fig. 2-1, before one millisecond.)

If the fireball growth rate defined in Eq. (2.2-2) is computed for the earliest time illustrated in the temperature profiles of Fig. 2-2, assuming for the moment an emissivity of unity, the rate is about 4.4×10^8 cm/sec. This rate is too high by an order of magnitude in comparison with results of the numerical calculation example. The calculation showed that the expansion at this 1.2 microsecond time was still being determined by radiation diffusion. The calculation, however, also treated the earliest times by diffusion, and not (as suggested above) by transport within a transparent heated region with a radius less than one mean free path for the emitted radiation. The appropriate mean free path for diffusion is the so-called Rosseland average, hereafter abbreviated as Rmfp. The Rmfp is defined as

$$\lambda_R \equiv \frac{\int_0^\infty \lambda_\nu \frac{dB_\nu}{dT} d\nu}{\int_0^\infty \frac{dB_\nu}{dT} d\nu} \qquad (2.2\text{-}3)$$

in which λ_ν is the spectral mean free path, B_ν (T) the Planck function, and $\int_0^\infty d\nu$ denoting integration over all frequencies. The Rmfp used in

the calculation approaches the Compton limit at high temperatures, however, and allows the rate of growth to be equally fast. In fact, without special consideration for relativistic effects, the diffusion growth can exceed even the speed of light.

At the earliest time illustrated in Figs. 2-2 and 2-3, the fireball has grown to more than one mean free path in radius which reduces the effectiveness of the inner temperature in driving the continued expansion. A more heuristic interpretation of the growth rate formula allows the emissivity to be interpreted as a resistance parameter which reduces the growth rate to less than the blackbody rate for that central temperature. An alternative interpretation treats this efficiency factor as one which compensates for the use of the shielded innermost temperature when the effective temperature is at some radial position further out and is lower in value, i.e., $e \approx (T_o/T_i)^4$ where T_o is the effective outer temperature and T_i is the screened central temperature. A crude measure of this correction and of an appropriate value for this viscosity constant might be the ratio of the Rmfp to the radius of the front, i.e., the reciprocal of the number of mean free paths between the radiating interior and the front. For the diffusion approximation, such a correction might better be expressed in terms of the local temperature gradient as well.

The inner temperature of our example calculation at 1.2 microseconds is around 10^6 ^oK (Fig. 2-2) and the density is still normal (1.29×10^{-3} gm/cm^3) (see Fig. 2-3). The Rmfp is a bit less than one meter, while the radius is about 3.2 meters. Taking e to be 1/3.2 brings the growth rate down to about 1.3×10^8, which is still high compared to that for the numerical calculation. The mean free path decreases rapidly as the temperature falls below 10^6 ^oK, however, and since the front at 1.2 μsec is at around half the interior temperature, a more appropriate mean free path might be between 0.92 (the value at 10^6 ^oK) and 0.12 (the value at 5×10^5 ^oK). Taking the average of their reciprocals, i.e., averaging the opacities, gives about 0.2, so that the correction factor, **e** , becomes 0.2/3.2, and the corrected rate becomes $\sim 3 \times 10^7$ which agrees well with the growth rate at that time from the detailed diffusion calculation.

The most appropriate specific energy and density values for use in the growth rate approximation are those just behind the front of the wave, since it is to those conditions that the cold air is to be heated, i.e., it is that heat capacity that will absorb the subsequent radiation energy flux. Fig. 2-4 displays the specific energy profiles for this one kiloton example for the same time as those of Figs. 2-2 and 2-3.

It is interesting to test the simple growth rate formula (Eq. 2.2-2) against the fireball growth speed that results from the numerical calculations. The calculation should show a rate faster than hydrodynamic shock growth until the radiation growth has fallen below the speed of hydrodynamic motions, and this simple form should show a comparable rate until that time, then a much slower rate as the shock wave takes over.

Fig. 2-5 compares these rates for the same time period as covered by the profiles of Figs. 2-2, 2-3, and 2-4 and beyond. In these comparisons, several approximations are represented by dashed curves, while the numerical calculation rate is shown as a solid curve. The rate calculated as blackbody at the inner temperature, shown as circled points, is clearly too high at all times. Even when the lower temperatures of the outer edge of the hot region are used to determine a blackbody rate (the triangles Δ of Fig. 2-5), the rate is at all times too high.

When the radiative resistance parameter is represented as the ratio of the Rmfp to the hot region radius, using the Rmfp evaluated at the hot interior temperature, the modified rate is still high at the early times when diffusion is still dominant (the square points of Fig. 2-5). It drops precipitously as the interior cools and becomes opaque at just the times when a shock begins to form (at about 10 microseconds in this example). Although this approximation is not correct in value, the sharpness of the decrease as hydrodynamics takes over can make it a useful indicator of the transition onset, and so a reasonable prediction tool.

The more accurate estimate of the early diffusion growth rate, involving the averaged opacity between interior and front, is also more subject to error due to the difficulty in judging appropriate front conditions. These estimates are indicated in Fig. 2-5 by diamonds. These values are closest to the numerical calculations rate of growth at the earliest times when diffusion is the dominant mechanism. The earliest profile front temperatures are difficult to define because the front is not sharp. The rate of growth estimated at these approximate front temperatures with

a corresponding resistance parameter leads to the estimates indicated by the triangles ▷ in Fig. 2-5. Again, the shock formation times is denoted by a sharp drop in the rate estimated in this manner. Both of the blackbody rate curves (upper curves of Fig. 2-5) show a fairly sharp drop at shock formation time. Such a simple but uncertain formula may be preferable to the use of the radiation resistance notion in determining shock formation radius and time. Since in this range of temperature and densities, the Rmfp decreases with decreasing temperature as about the fourth power of the temperature, using the Rmfp as a correction factor then means that the adjustment parameter is as sensitive to temperature changes at the blackbody rate itself. Such critical opacity dependences may provide some sharp distinctions in estimates but at the same time present some hazards in choosing effective temperatures too casually.

After shock formation, the rate of growth of the fireball should follow the shock growth itself until the shock cools to transparency. The shock speed for a strong shock is approximately given by

$$\dot{R}_s \sim \sqrt{\frac{(\gamma_s+1)P_s}{2\rho_o}} \qquad (2.2\text{-}4)$$

where $\gamma_s = (P_s/\rho_s E_s)+1$, ρ_o is the ambient air density and P_s , ρ_s , and E_s are shock front values of pressure, density and internal specific energy. This approximation is shown in Fig. 2-5 by the symbol ◁ . For the earliest times, the expansion is faster than this shock rate, but at later times it corresponds well.

Using the particle velocities (u_s) at the front and the density at the front (through the mass conservation relation) provides the relation

$$\dot{R}_s = \frac{u_s \rho_s}{\rho_s - \rho_o} \qquad (2.2\text{-}5)$$

The rate derived from corresponding values of u_s and ρ_s for the numerical calculation is indicated in Fig. 2-5 by triangles pointing down (∇). After nuclear shock catch-up this curve coincides with the solid curve for the directly computed rate.

In the temperature region of interest, a shock can be pictured as a sharp gasdynamic jump imbedded in a region of radiation-induced temperature variation (Fig. 2-6). The internal structure of this type of wave has been investigated extensively by Zeldovich (1957), Raizer (1957), and Heaslett and Baldwin (1963), to name a few, all of whom employed the equations of steady continuum gas dynamics with gray radiative transport.

The important feature of this picture is the temperature precursor which runs ahead of the sharp front. This precursor is created by the radiation from the high temperature region behind the sharp front. One can estimate the temperature of the precursor by equating the power radiated by this front with the rate of heating in the precursor. In the resulting relation

$$\rho u e_p = \sigma T_s^4 \qquad (2.2\text{-}6)$$

ρ, u and e_p stand for the ambient air density, the shock velocity and the internal energy of the air in the precursor. From the latter quantity and the equation of state, one can then obtain the temperature of the precursor. Using the Hugoniot relations (Section 5.1 of (4)) and a simple analytic fit

to the equation of state, one obtains at sea level the relation

$$T_p = 3.45 \times 10^{-7} \, T_s^{2.17} \qquad (2.2\text{-}7)$$

For a shock temperature of 10^5 ^oK we calculate a precursor temperature of $23,000^oK$ and note that a millimeter layer of air at that temperature is opaque. Up to the time when the shock temperature drops to 10^5 ^oK all the thermal radiation comes therefore from the precursor. To make a quantitative evaluation of the power radiated during this phase requires a more detailed analysis of the radiative transfer problem. Qualitatively one can see that the power must decrease with time and this is the decrease following the early peak in Fig. (2-1).

As the shock temperature drops below 10^5 ^oK, the precursor cools to where it gradually becomes transparent so that the radiation from the shock front begins to shine through. When this happens the power-time curve goes through the minimum which is shown in Fig. 2-1 as the shock precursor minimum (SPM). While the shock front gets more and more exposed, the power rise because of the exposure is eventually compensated by the temperature drop of the shock itself and at that time the power level reaches the maximum which is shown on Fig. 2-1 as the shock exposure maximum (SEM).

During the phase following this maximum the rate of thermal radiation loss from the fireball can be characterized as that from a blackbody sphere at the shock front temperature and of radius equal to that of the shock radius. Although such a rate describes the fireball emission, the power observed at any distance will contain only that fraction which the cold air

outside the fireball is capable of transmitting. To a good approximation, that fraction can be calculated by assuming a simple cut-off in the transmitted spectrum. Values of this fraction $f(T_s,\nu_c)$ where ν_c is the frequency corresponding to a cut-off at 1860Å (representing the edge of the O_2 absorption) are shown as functions of the temperature (T_s) in Fig. 2-7). The fraction is evaluated from a tabulation of the Planck radiation function and its partial integral by Gilmore (1956). The fraction is defined as

$$f(T_s,\nu_c) = \frac{15}{\pi^4} \int_0^{x_c} \frac{x^3 dx}{e^x - 1} \qquad (2.2\text{-}8)$$

where $x_c = h\nu_c/kT_s$ and h and k are Planck's and Boltzmann's constants, respectively ($h \approx 6.625 \times 10^{-27}$ erg sec, $k \approx 1.380 \times 10^{-16}$ erg/^oK).

During this phase which lasts until the shock temperature has dropped to so low a value as to make the shock front transparent, the following simple expression characterizes the thermal radiation rate for an air burst nuclear explosion:

$$P \approx 4\pi R_s^2 \sigma T_s^4 f(T_s,\nu_c) \quad , \qquad (2.2\text{-}9)$$

in which R_s represents the shock radius, T_s the temperature, σ the Stefan-Boltzmann constant (5.672×10^{-5} erg/cm^2/deg^4/sec) and $f(T_s,\nu_c)$ the fraction passed by the cold air.

Unit optical depth for most frequencies grows longer as the shock front cools, so that emission from hotter air behind the front begins to shine through. The shock front itself becomes fainter and appears to pull ahead of the luminous fireball, a phenomenon which is referred to as the

"breakaway" (Gladstone, 1962, Section 2.110). Because the shock has been carrying the shock-heated air outwards with its expansion, a rather steep gradient in temperature is maintained just behind the front, so that a slight increase in unit optical depth exposes higher temperatures but at no appreciable decrease in radius of effective radiating surface. At this time the power curve goes through the principal minimum (PMIN) in Fig. 2-1.

Fig. 2-8 indicates the geometry of fireball temperatures (in cross-section) at a time somewhat beyond the time of minimum thermal power. While the thermal radiation increases, and while progressively deeper parts of the fireball are exposed, the hydrodynamic expansion dominates so that the visible or apparent fireball size continues to grow. Eventually, the luminous fireball stops expanding and the power output reaches the final maximum (FMAX).

Throughout this radiative and then hydrodynamic expansion of the fireball, right up to the time of minimum light intensity, something less than half of one percent of the total yield has been radiated out of the fireball. Both integrals of the measured power-time data from tests and of the simple expression given above for radiation from the fireball (as determined by shock front conditions) lead to an answer close to 0.44%. In the latter integral, the properties of the shock front are sufficiently well defined by almost any calculation - even those not accounting for radiation transport in the early phases, but necessarily taking account of the real gas properties of air. (e.g., Brode, 1956a,b).

Since the air just behind the shock is much hotter and much less dense than the air at the front itself (see Figs. 2-2, 2-3, and 2-4), the rate of thermal radiation increases rapidly when that air is exposed, until the hottest temperatures at the back of the steep gradient

behind the shock front become visible and are radiating directly to the exterior. Thereafter, as the size of the radiating sphere shrinks and the interior cools, the rate decreases. This is the period in which the fireball history comes closest to the cooling wave notion expressed in a simple form by Zel'dovich, Kompaneets and Raizer (1958) and applied into a fireball theory by Bethe (1964). The notion is that a recognizable and fixed form cooling wave erodes the hot fireball interior, beginning at the exterior and working inwards. After the shock front has become transparent, such a cooling wave process is very likely operating, but it is not at first working into a fixed or uniform temperature or density, and it is not shrinking the fireball. The outward hydrodynamic expansion is still too strong. When the outer regions have all become sufficiently cool and transparent so that the inner radiation-heated region is exposed, then the conditions suggested for a cooling wave are approximated. Even then the temperatures are not constant and the surface area is shrinking rapidly, so that the cooling rate decreases. When this interior sphere has cooled to below about $10,000^{\circ}K$, the whole of the fireball has become relatively transparent, and the subsequent radiation losses are characterized more by a grey body approximation, i.e., characteristic of a volume of air of low emissivity - one of less than unit optical thickness. It may also still be expanding adiabatically, and contributing energy to the shock growth.

Temperature profiles spanning this period from principal minimum through final maximum and on to a transparent fireball are illustrated in Fig. 2-9. For a yield of one kiloton, the cooling wave is less obvious as a wave than as a rather sudden depletion of the hottest interior region. At larger

yields, where more optical thickness is represented at every stage, the progress of a cooling wave from outside toward the center is more easily imagined (Brode, **1964a, Fig. 5, b Fig. 15).**

In this rather complex power radiated history of two or three maxima, as illustrated in Fig. 2-1, the final pulse represents a total energy of 30 or 40% of the total yield of the nuclear device. When all the energy is accounted for , including that in the infrared which originates in shock heated air outside the visible fireball and is radiated only very slowly, the fraction may be even larger.

There are several features of this one kiloton explosion that have not yet been mentioned and that are of lesser influence on the thermal radiation and fireball behavior at sea level, but which become relatively more important at other yields or altitudes. One such feature is a second shock wave which originates within the bomb vapors, traverses the early sphere of hot air behind the radiation front, and overtakes the strong shock that forms the fireball surface at later times. This debris or bomb shock is seldom in evidence in sea level explosions, and has lost most of its energy long before it overtakes the main shock, so that it contributes little to the fireball surface or thermal radiation histories. Because the hot interior of the fireball is for most of the fireball expansion a region of long mean free path, it is a region of nearly uniform temperature. When the case shock compresses and heats this air further, some of that heat is promptly re-radiated ahead, forcing this interior shock to behave isothermally rather than adiabatically. This isothermal shock can lose energy very rapidly by this means, and may persist only through the

continuation of its outward momentum.

A history of the radii of this shock and other fronts in this kiloton example is shown in Fig. 2-10. When this debris shock travels outward to the edge of the fireball, it encounters a sharp discontinuity in density. At that point, a reflected shock originates and is returned inward to implode upon the origin. Here again, is a phenomenon which has no consequence for this example, but may be prominent in high altitude events. The vaporized bomb expands along behind this debris shock, but at sea level is not visible until very late - after the second maximum in the thermal power. This bomb debris is not realistically treated in any of the usual calculations, since they invariably assume radial symmetry and allow no mixing or turbulent flow. When it emerges in the transparent fireball at late times, the vaporous debris has become highly turbulent and has evidently mixed with considerable fireball air.

Although Fig. 2-10 indicates a transition from radiation expansion to strong shock expansion, the radiation diffusion does not stop. As the shock brings down the density in the interior air, the opacity of that air decreases also, and the radiation is allowed to diffuse into some of the now shock-heated air. The dotted curve below the shock front curve of Fig. 2-10 indicates the position of the radiation front. Most of its outward excursion is due to the flow of air in the expansion behind the shock itself. At times later than shown in Fig. 2-10, the radiation front and the visible fireball drop behind. The short dashed curve near the end of the shock front curve of Fig. 2-10 represents a position close to the fireball front - being the locus of points at 5000°K - with higher temperatures inside of that radius, and colder temperatures outside.

The continued flow of radiation is made more obvious in a plot of the temperature histories of several shells of gas representing the air that was shocked to a particular temperature, cooled in the subsequent adiabatic expansion, but then reheated by the radiation wave following. Such a set of curves are shown in Fig. 2-11, where at particles shocked to 10^5, 70,000 and 40,000°K the adiabatic cooling is arrested by the arrival of the radiation diffusion wave which causes that shell of air to rise in temperature again. The air starting at the 20,000°K shock point is never over-run by the radiation wave, i.e., the radiation wave stops before it gets that far, having run out of energy and not being aided by further expansion which would help to reduce the opacity of the cooler air in front of it.

A great many nuclear weapon applications, tests, and effects interests involve the thermal and fireball effects of nuclear explosions on or close to the surface of the earth. Many interesting and novel interactions occur which are not evident in air bursts well away from the surface. However, there is no intention of providing a review of these factors in this current effort. It should suffice to point out that all of the essential features which are described and followed here are also an important part of surface bursts, while the latter are further complicated by the early injection into the fireball of massive amounts of earth material, and by the geometric distortions of the fireball that occur as a consequence of shock and thermal reflections from the earth's surface. The change in radiator shape from spherical to at best hemispherical or worse a partially obscured hemisphere means that the thermal flux to other points on the earth's surface will be less than that from an air burst. Total flux at

points in the air above the burst may at the same time be increased.

As the earliest pictures of nuclear explosions (Glasstone, 1962) clearly show a further consequence of the ground involvement is the "dust skirt" which precedes the fireball shock and largely obscures the base of the fireball. Although not visible in any of the pictures, there must also be vast amounts of earth shovelled into the hot fireball interior at an early time (Brode and Bjork, 1960), and this material cannot fail to have profound effects on both the temperature and thermodynamic state of the fireball gases and on the opacities or optical properties of that region. Test observations indirectly attest to the influence of such surface effects.

Observations and measurements at very late times in the fireball history show that the radiation rate trails off with a very long tail (as in Fig. 2-1) and comes from shapes other than simple spheres. The fireball at late times is like a bubble in the atmosphere - having very low densities in its interior - and so it rises, and in rising breaks up at the bottom to transform itself into the familiar atomic cloud ring or torroid which rolls its way up through the atmosphere. The torroidal circulation that is induced is quite strong and serves to severely limit mixing of the hot fireball gases with the exterior cold air, thus prolonging the existence of air and debris at temperatures of thousands of degrees Kelvin, while the cloud rises in the atmosphere. When much earth material and/or water vapor is present, the late fireball remains opaque, and the rate of late radiation is more determined by the rate of turbulent mixing which brings hot gases to the cloud surface rather than by the radiation transport

properties alone. For an air burst well above the surface, however, the late fireball becomes quite transparent, so that only a faintly luminous ring assures us that the rise and circulations are much the same as for lower bursts.

2.3 Other yields and altitudes

The example of a one kiloton detonation at sea level contains all the basic physical phenomena which enter into consideration at other yields and at altitudes up to about 70 km. The overall appearance is, nevertheless, appreciably different since the individual events which are responsible for the various maximum and minima in Fig. 2-1 occur at different times.

In carrying out a discussion of these changes, it is useful to note that the relation between shock radius and time can be approximately represented by a hydrodynamic scaling law. To formulate this we introduce the scaled variables

$$R^* = \left(\frac{\bar{\rho}}{Y}\right)^{1/3} R \qquad (2.3\text{-}1)$$

$$t^* = \left(\frac{\bar{\rho}}{Y}\right)^{1/3} t \qquad (2.3\text{-}2)$$

where Y is the yield of the explosion and $\bar{\rho} = \rho/\rho_o$ the ambient air density relative to that at sea level. In our 1 kiloton sea level example the scaling factors are of course unity.

The scaling law is not valid until after the debris shock has caught up with the somewhat slower shock which is driven by only a fraction of the total yield. The scaling law can be deduced from the strong-shock solution for a point source (Taylor, 1950; and Sedov, 1959). This limits the validity of the scaling laws at late times when the shock becomes weak. The law obtained in this manner takes the form

$$R_s^* = k(t^*)^{2/5} \tag{2.3-3}$$

where the subscript s denotes that the value is taken at the shock front.

From the radius $R_s = 20$ m read off Fig. 2-10 for $t = 1$ msec, one determines the proportionality factor to be

$$k = 20 \text{ m } (\text{msec})^{-2/5} \quad (\text{KT})^{-1/5} \tag{2.3-4}$$

In the above scaling law the scaling factors cancel out of the expression for the shock velocity

$$\frac{dR_s}{dt} = \frac{dR_s^*}{dt^*} = \frac{2}{5} k(t^*)^{-3/5} \tag{2.3-5}$$

which makes this velocity a function of t^* only. Applying the Hugoniot relations one can now show that the temperature T_s behind the shock is

also very nearly a function of t^* only. This is not an exact result because it depends on certain assumptions about the equation of state which are only approximately true. If one checks the prediction that T_s is a function of the scaled time only against computed results, one finds that it fits the changes with yield at a given altitude very well. The changes with altitude at a given yield are not given with quite the same accuracy, but are still sufficiently close for most purposes.

It should be noted that the above scaling procedure differs somewhat from the so-called Sachs scaling where one introduces the variables

$$\tilde{R} = (p/Y)^{1/3} R \qquad (2.3\text{-}6)$$

$$\tilde{t} = (p/Y)^{1/3} (p/\rho)^{1/2} t \qquad (2.3\text{-}7)$$

If one expresses the ambient pressure and density p and ρ, the yield Y and the variables R and t all in the same system of units,the scaled variables are dimensionless. It is more convenient, however, to replace p and ρ by the ratios $\bar{p} = p/p_o$ and $\bar{\rho} = \rho/\rho_o$ relative to the sea level values, and to express Y as before in KT. With this choice, the strong shock relation between $\tilde{R}$ and $\tilde{t}$ is the same as between R^* and t^*, i.e. Eq. (2.3-3) with the same value of the constant k.

The two methods of scaling differ in regard to what are considered similar situations. For the starred variables similarity implies that for example the hydrodynamic velocity and the temperature are unchanged; for the variables with the tilde the Mach number and the temperature ratio T/T_o are unchanged. Either choice is acceptable, but ours has the advantage of using only one parameter to characterize the altitude.

From the time of shock formation until breakaway the thermal radiation comes partly from the shock precursor and partly from the shock front, and it is evident that the shock temperature is a major factor in determining the timing of the maxima and minima during this period. At a given altitude where one has a one-to-one relation between shock and precursor temperature (see Eq. 2.2-7 for the sea level case) it is fairly accurate to state that the shock formation maximum, the shock precursor minimum and the shock exposure maximum occur at fixed values of the shock temperature and therefore at fixed values of the scaled time. As one considers different altitudes the relation between T_p and T_s changes and one finds different values of the scaled times associated with these features of the power curve.

After breakaway the radiation comes from points to the inside of the shock front whose locations depend on the optical properties of the air and in turn on the temperature and density distribution. This is a radiative transfer problem and hydrodynamic scaling, where times vary as the cube root of Y, is replaced by radiative scaling, where times vary approximately as the square root of Y (Glasstone, 1962, section 7.92).

Altitude scaling is a more difficult problem than yield scaling. We have already mentioned the effect of the changing relation between T_s and T_p. To this we must add that the relative importance of hydrodynamics and radiation transfer shifts with increasing altitude in favor of the latter. Thus shocks form more slowly and radiation is emitted more rapidly as one goes to higher altitudes. As a result the features before breakaway are increasingly delayed and the maxima and minima tend to become weaker. The final radiative pulse on the other hand advances in time and becomes more prominent. At about 50 km the early features have become washed out and what was the final pulse is now the only pulse.

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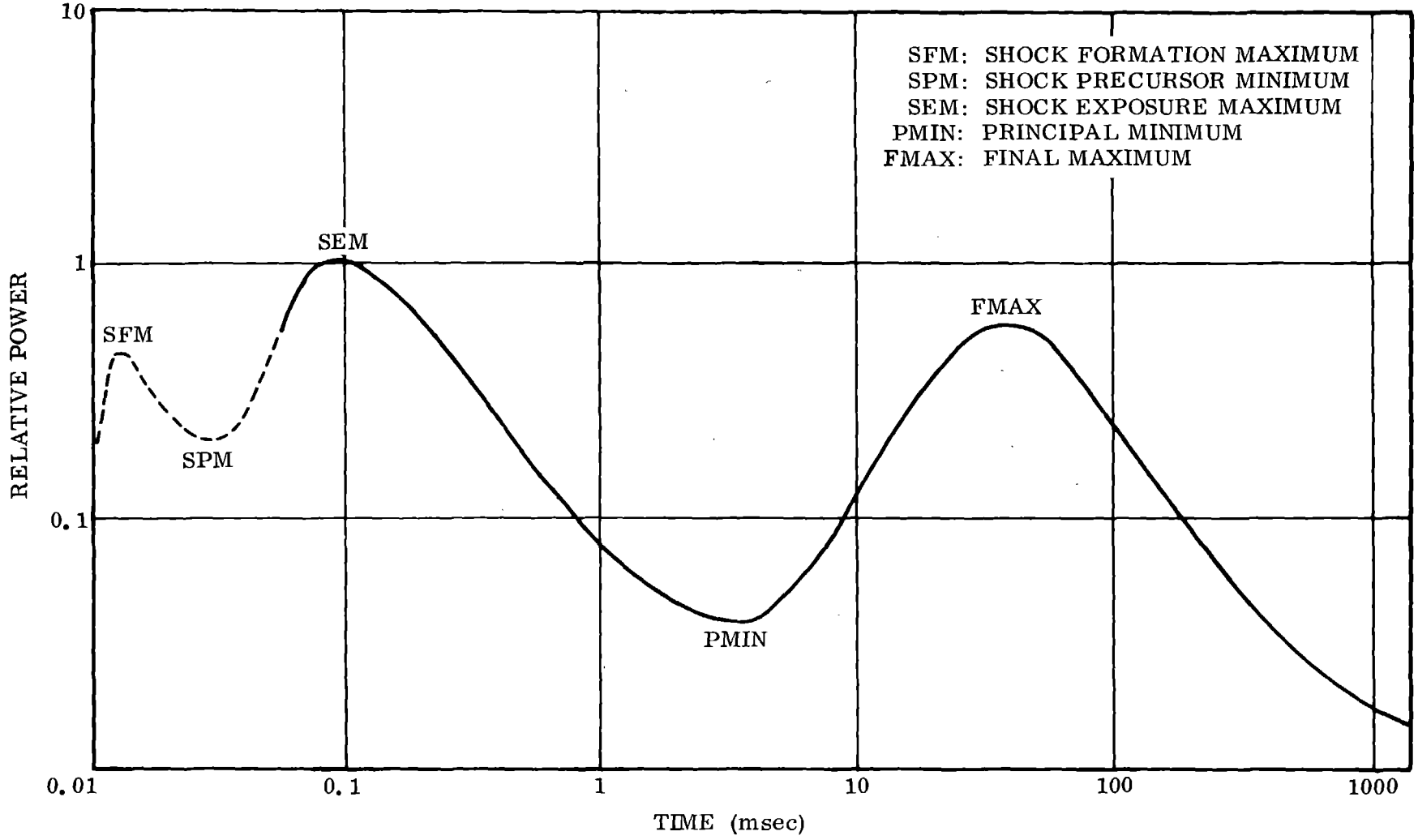

FIG. 2-1 THERMAL POWER RADIATED FROM ONE KILOTON AT SEA LEVEL

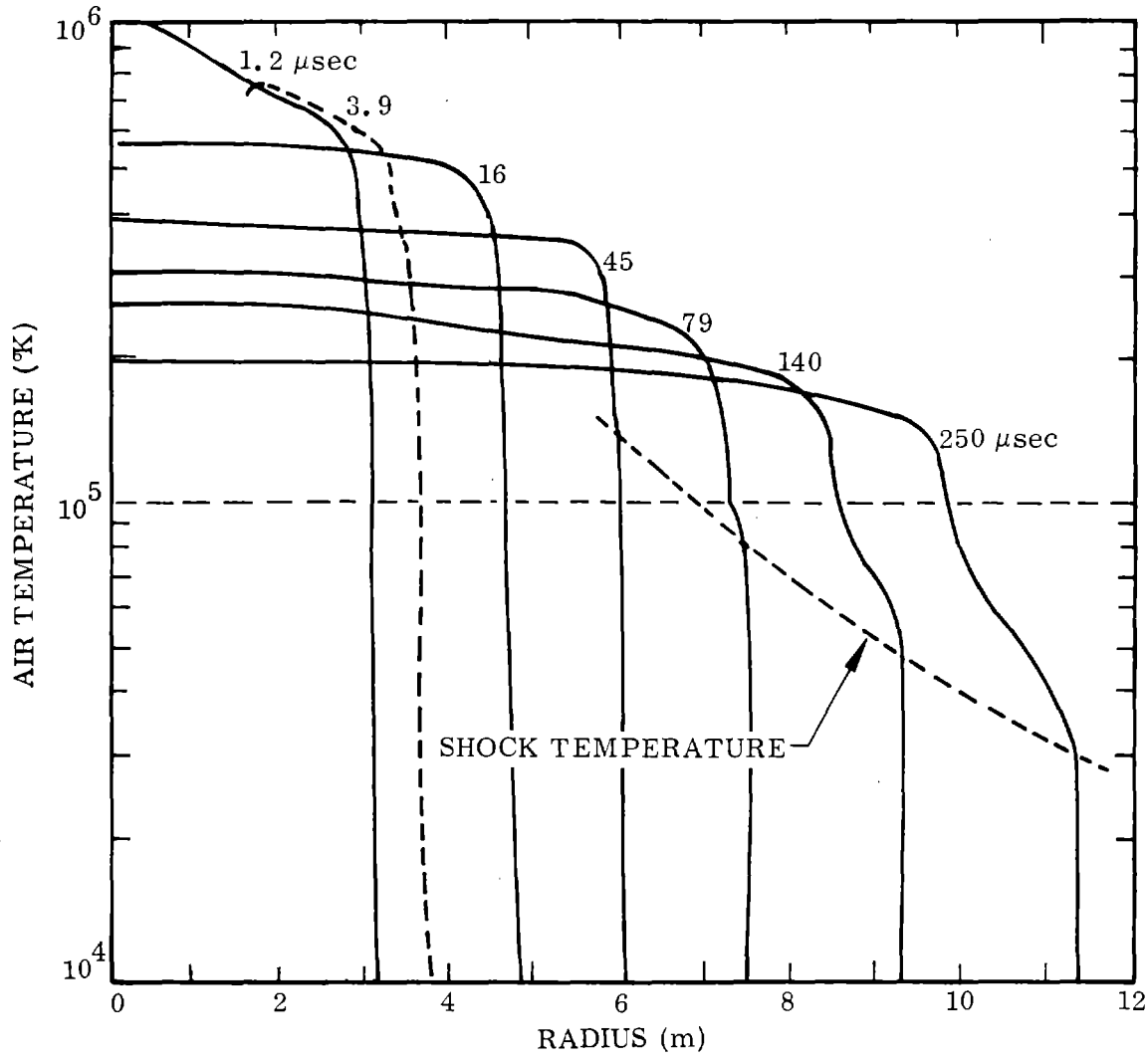

FIG. 2-2 EARLY TEMPERATURE PROFILES - 1 KT, SEA LEVEL

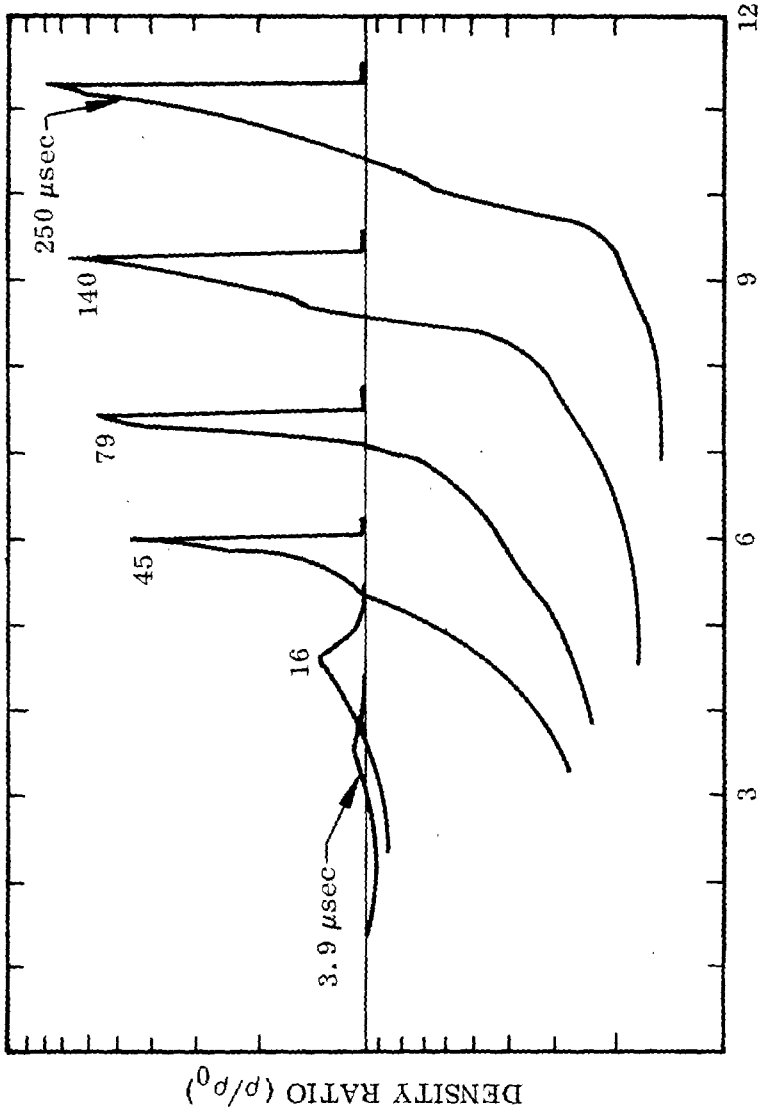

FIG. 2-3 EARLY DENSITY PROFILES - 1 KT, SEA LEVEL

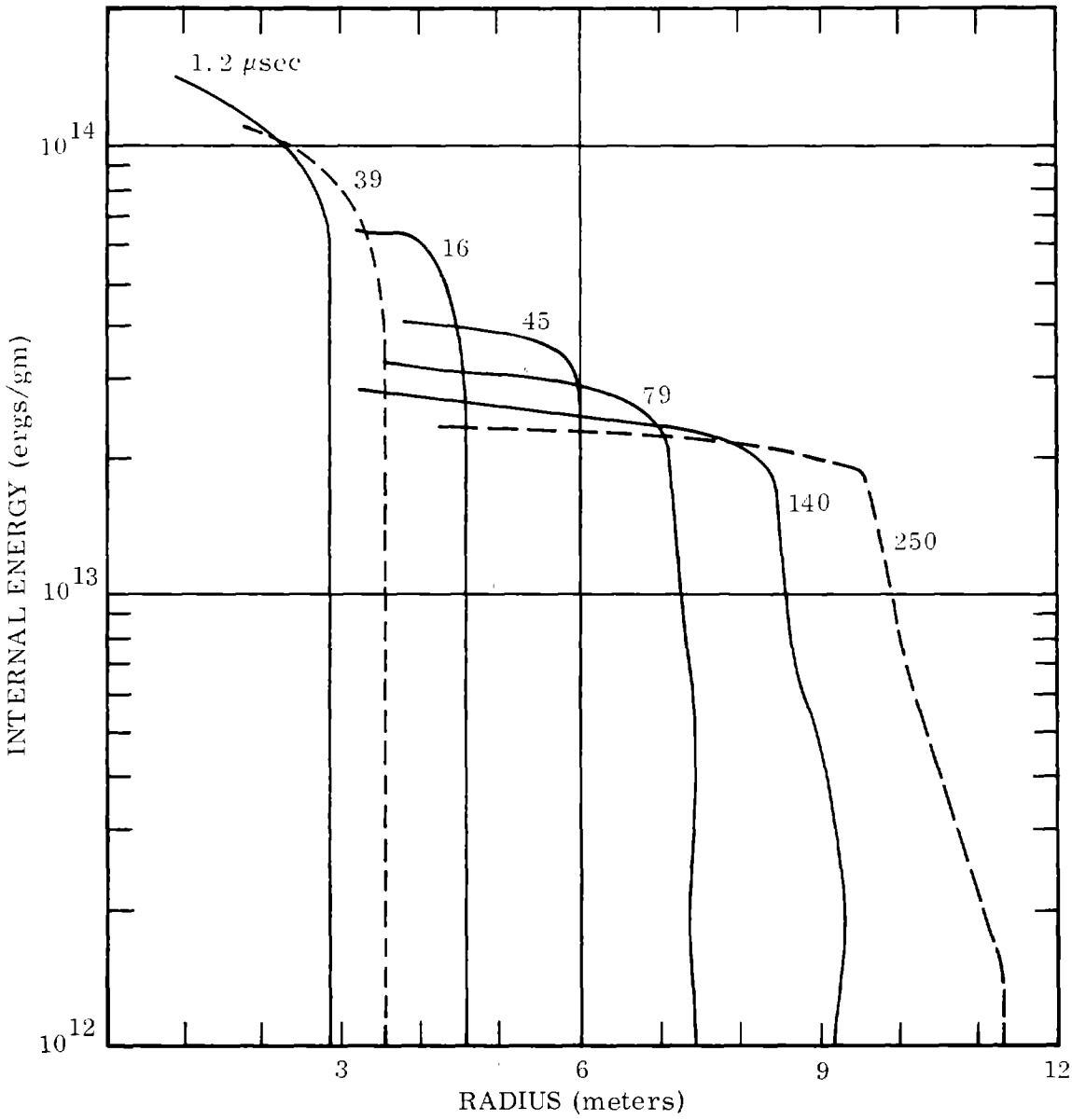

FIG. 2-4 EARLY ENERGY DENSITY PROFILES - 1 KT, SEA LEVEL

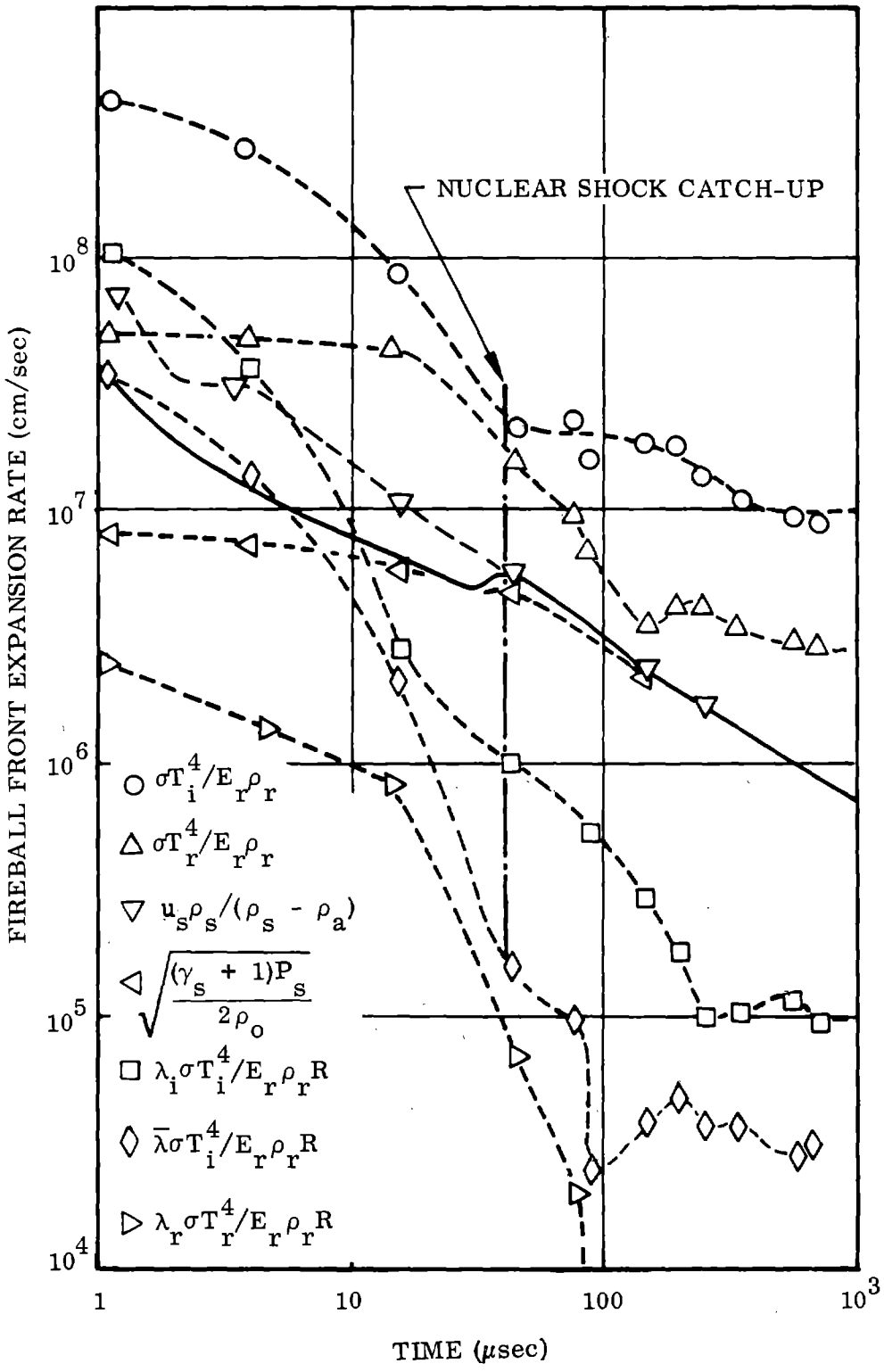

FIG. 2-5 COMPARISON OF VARIOUS APPROXIMATE RATES WITH EARLY FIREBALL GROWTH RATE

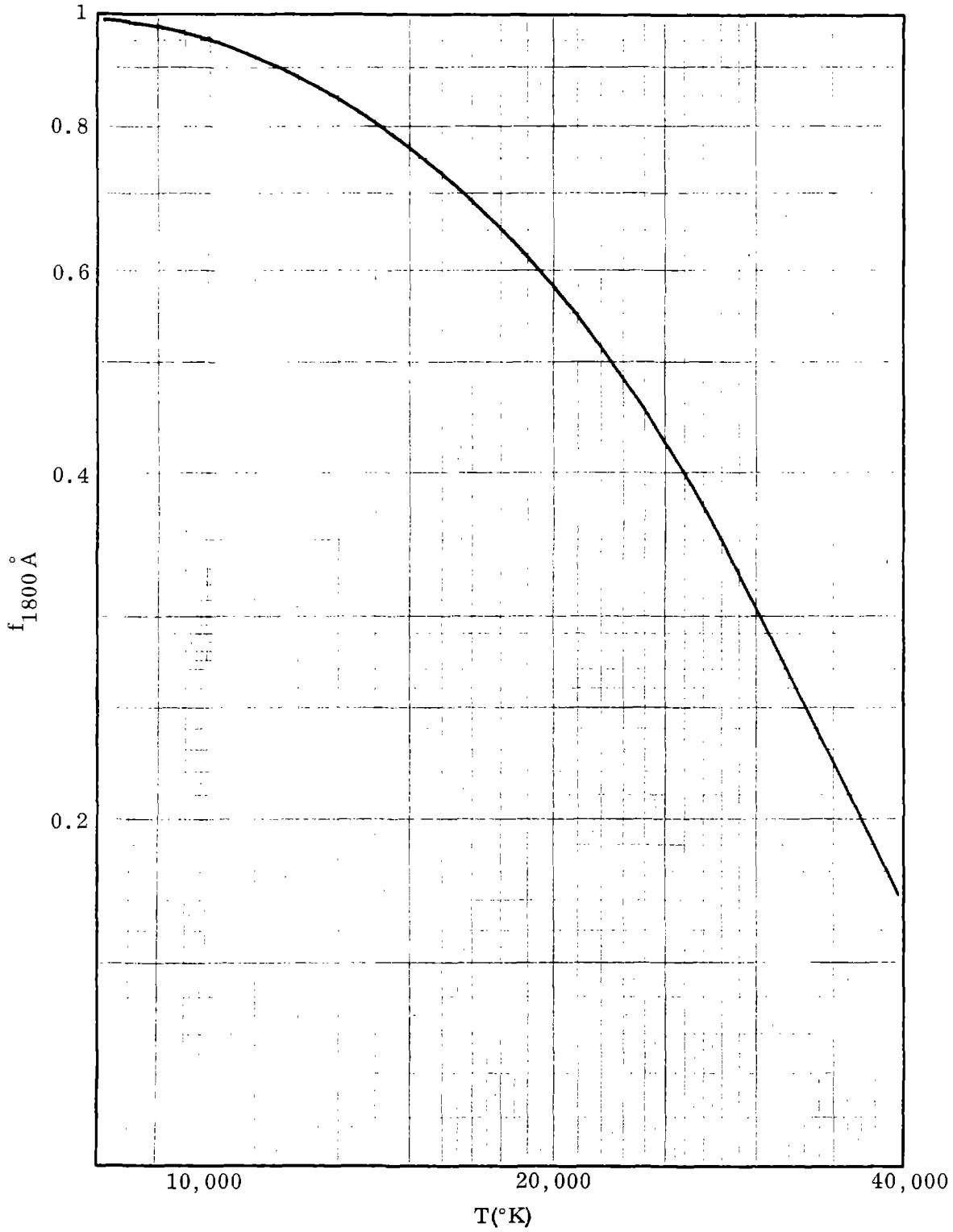

FIG. 2-6 FRACTION OF BLACKBODY RATE AT WAVELENGTHS LONGER THAN CUTOFF AT 1860Å

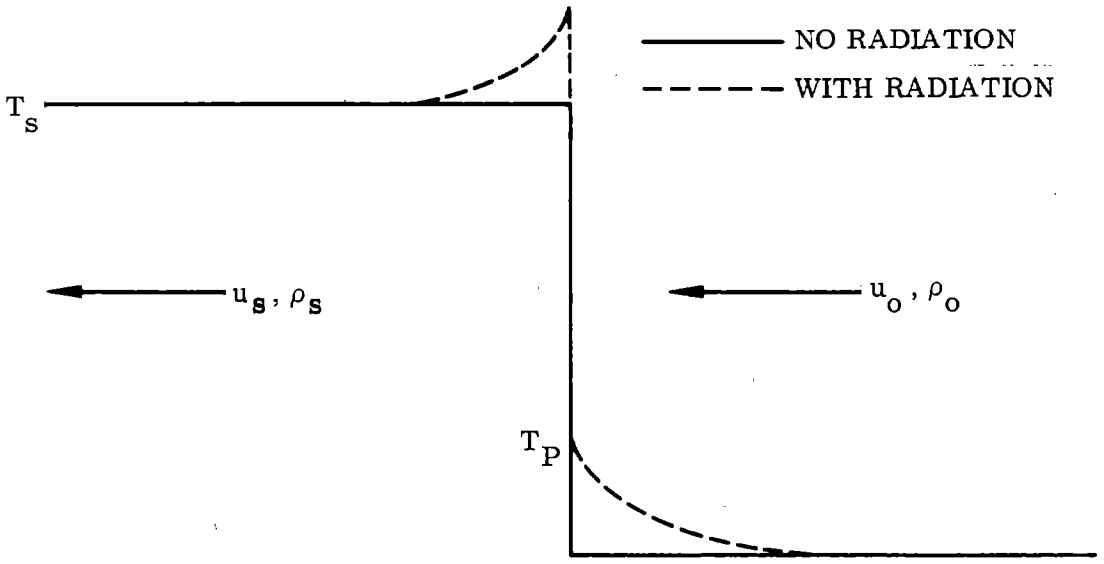

FIG. 2-7 SHOCK TEMPERATURE PROFILES WITH AND WITHOUT RADIATIVE TRANSPORT

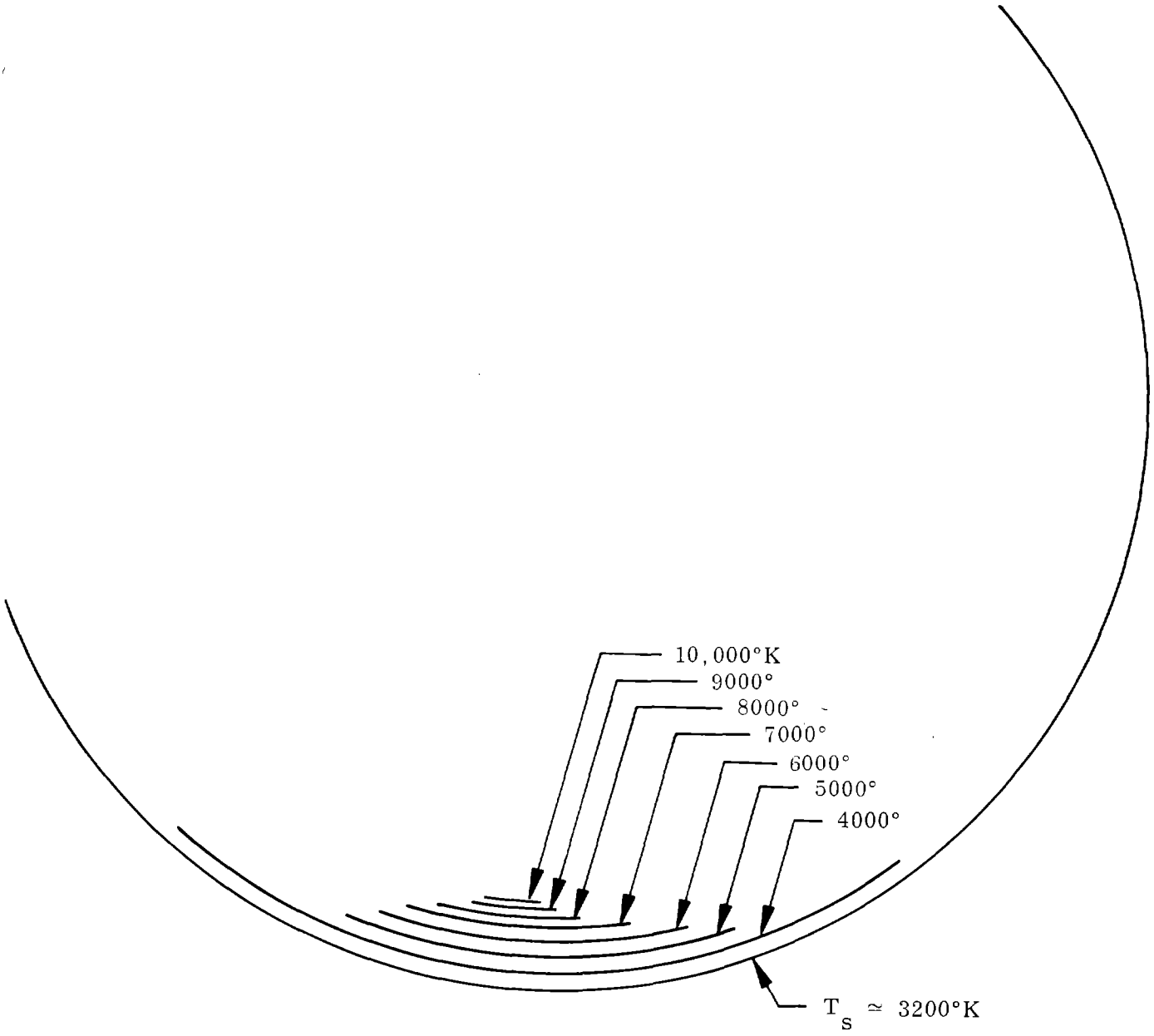

FIG. 2-8 FIREBALL THERMAL GRADIENTS AT TIMES NEAR MINIMUM LIGHT INTENSITY

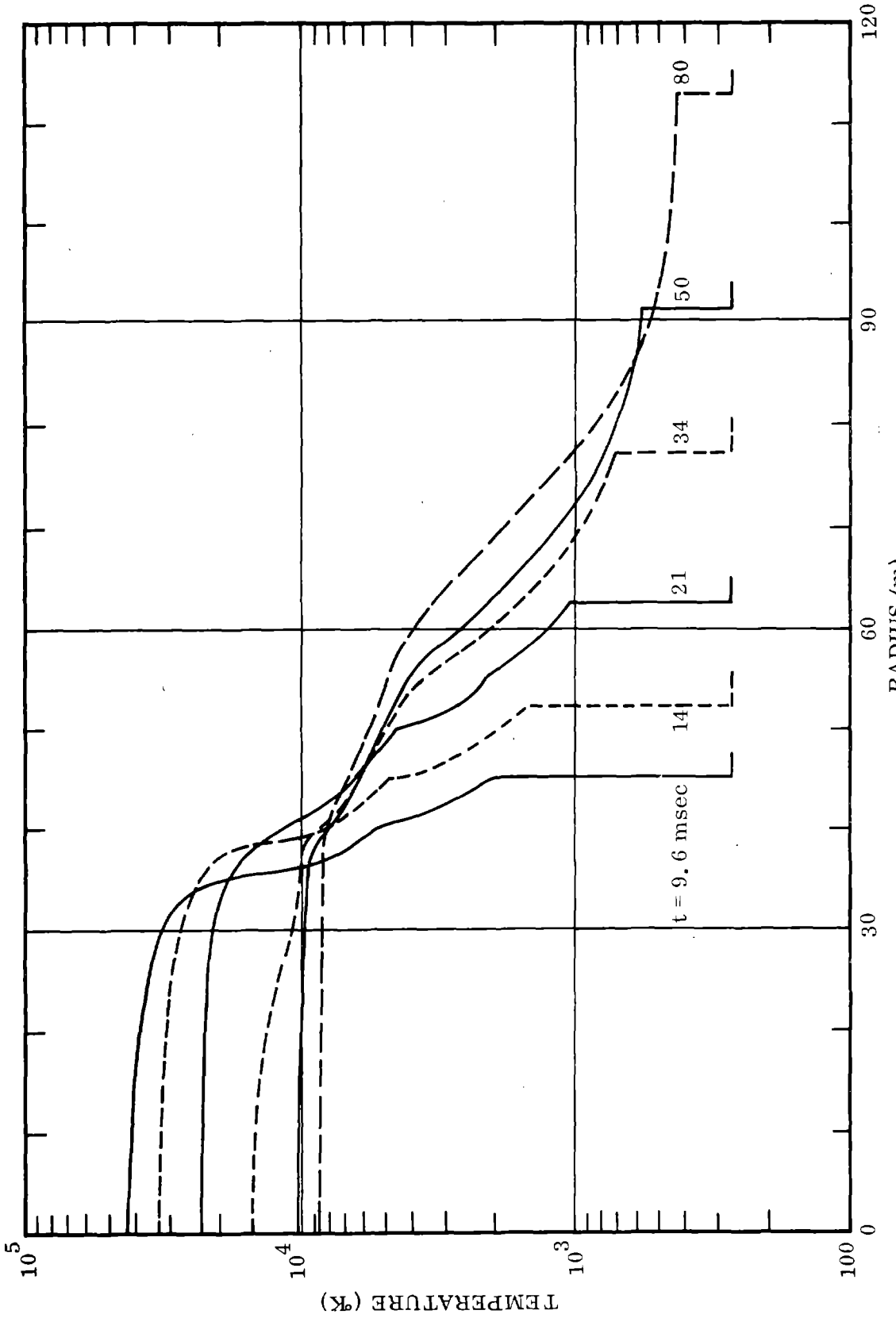

FIG. 2-9 LATE FIREBALL TEMPERATURE PROFILES - 1 KT, SEA LEVEL

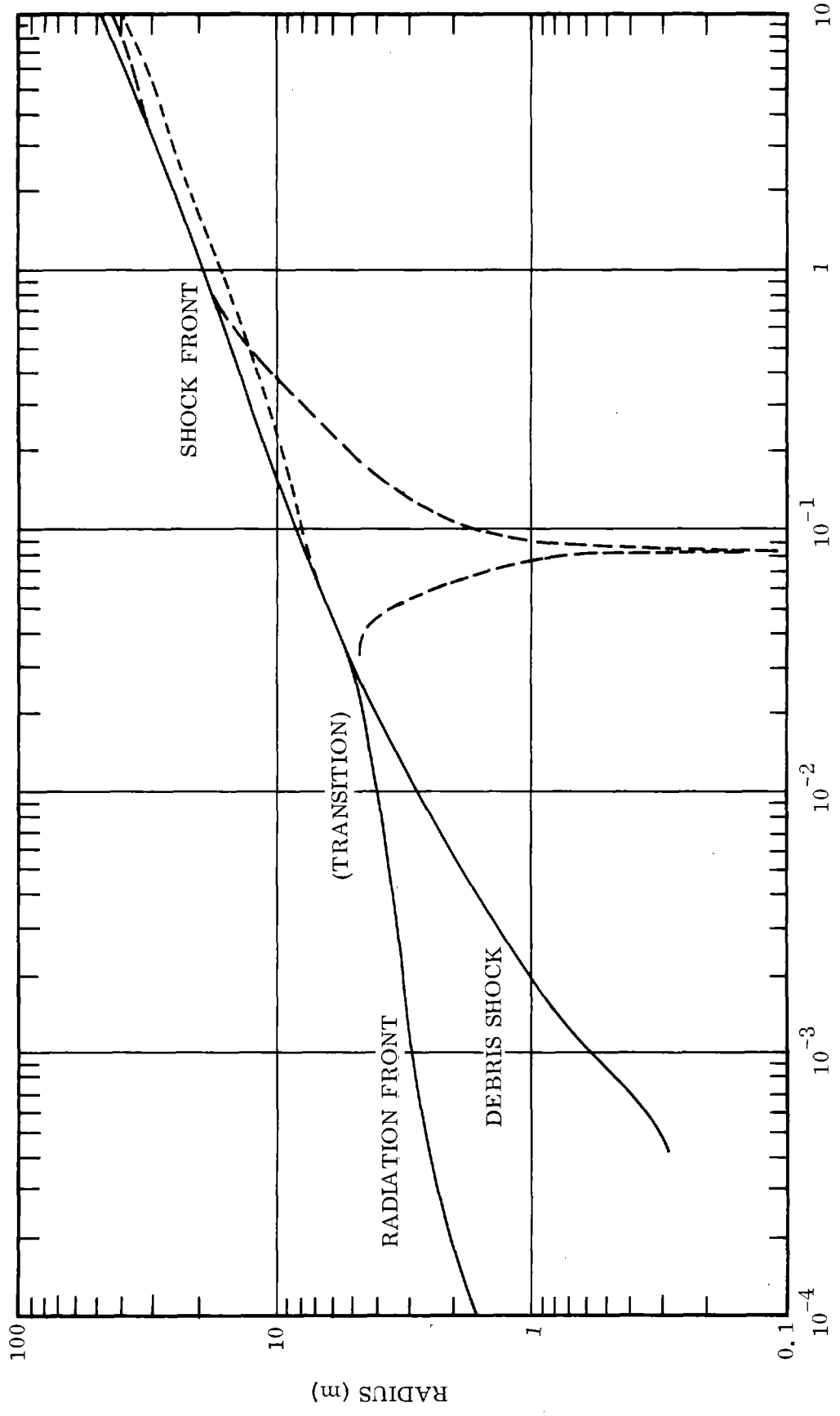

FIG. 2-10 EARLY RADIUS-TIME FOR A ONE KILOTON - SEA LEVEL FIREBALL

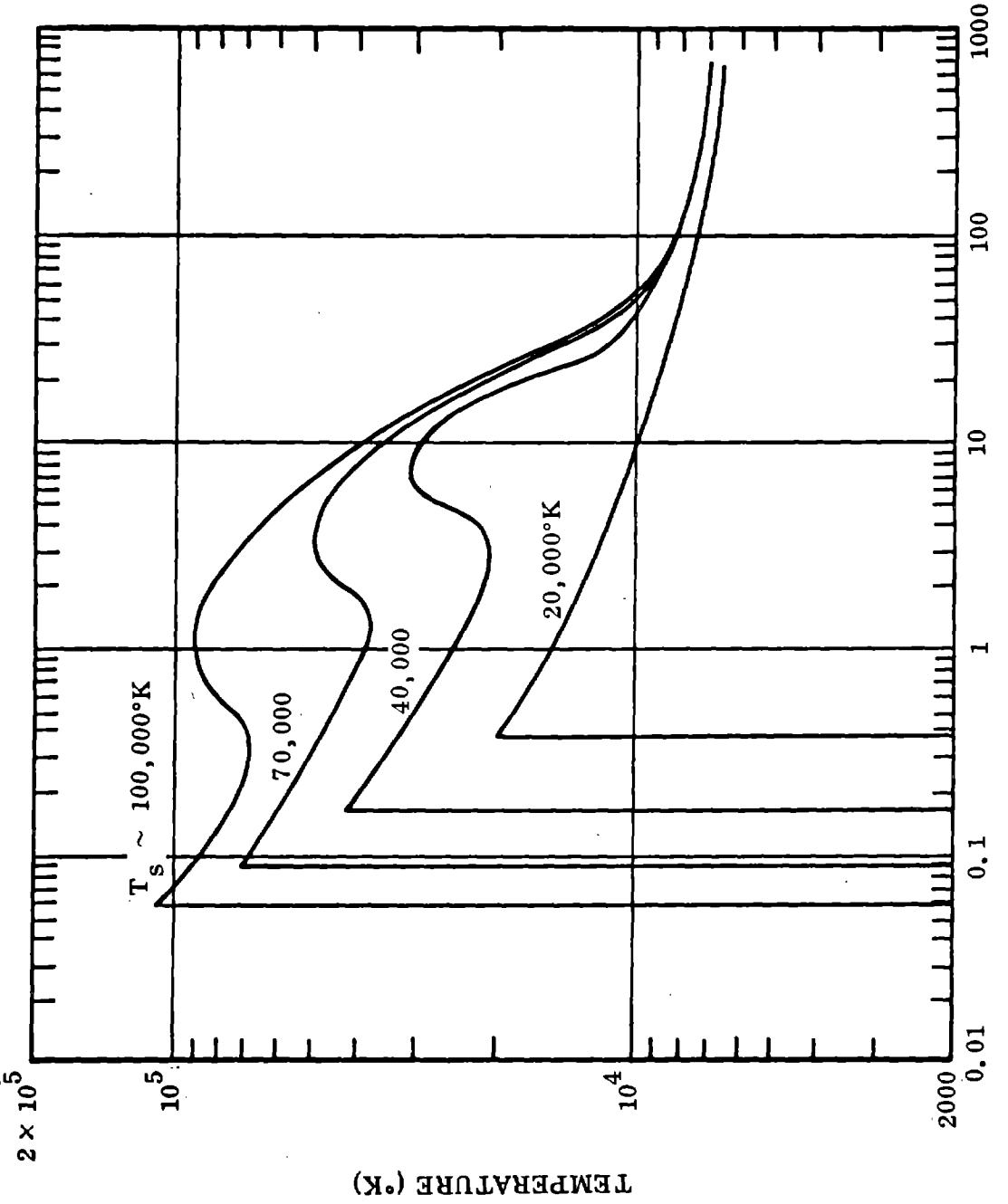

FIG. 2-11 AIR MASS TEMPERATURE HISTORIES - SHOWING ADIABATIC EXPANSION COOLING AND THE HEATING BY THE FURTHER GROWTH OF THE RADIATION DIFFUSION WAVE

Chapter 3. SUMMARY

As the reader of this report will have gathered any attempt at following the evolution of a fireball by numerical means utilizes a whole spectrum of facts and assumptions ranging all the way from being undisputable to being highly suspect. This is likely to leave him with a somewhat uncomfortable feeling about the reliability of such a calculation. In this summary we will put the finger on some of the underlying assumptions, point out what we know about their validity and evaluate how strongly our lack of basic information or of the willingness to spend computing dollars will influence the final product.

3.1 Equation of state

Nearly all calculations make the basic assumption that the air remains throughout in a state of LTE.* Once this is accepted it follows that the relation between the various state variables can be found by the methods of statistical mechanics. The application of these methods is very straightforward and the results as presented in (1) and (3)** are probably correct to within a few percent. In some instances analytic fits which were made to feed these results into a computer have been poor but this problem can certainly be overcome and should not contribute significantly to errors in hydrodynamics or other phases of the main calculation. Some problems may arise in the central region where one has debris rather than air and even more so in the transition region where one may have a debris air mixture. Fortunately many important results are rather insensitive to these details.

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* Local Thermodynamic Equilibrium (LTE).

** (1) stands for DASA-1917-1, etc.

3.2 Absorption coefficients

This subject has been discussed in detail in (2) and related facts are brought up in Chapter 1 of this volume. There are several ways of describing the absorption which differ in the amount of detail which is presented. The most detailed description consists of a listing of lines with intensities and line shape parameters on top of a continuum. All these factors are subject to errors as we shall briefly discuss.

In the low temperature case where the lines are due to molecular systems the information comes largely from experimental spectroscopic studies. The limitations of our knowledge about frequencies and intensities is discussed in (2) Chapter 7. The information on line shapes is almost non-existent. To this one should add that one can hardly afford to include any but the strongest band systems. Even the rather minimal choice of eight band systems in the most recent version of the SACHA program brings the number of transitions to over 190,000.

In the high temperature case the absorption comes from inverse Bremsstrahlung and from transitions in atoms and atomic ions. There is a strong continuum due to the first and due to photoionization. There is a fairly well developed theory and some experimental information backing it up. On top of the continuum is a large number of lines. A few levels have been observed experimentally but the majority, especially for the highly ionized atoms, have not been observed and must be obtained theoretically. It is certainly necessary to find the transition probabilities by quantum mechanical methods. These are so complex that one is forced to make radical approximations to get any answers and the results are not very reliable.

The calculation of the line contribution is the most elaborate part of the program and again one can hardly afford to include any but the strongest lines. This involves a somewhat arbitrary cut-off procedure whose practical effect can only be evaluated when one specifies how the absorption coefficient is to be used.

The detailed description of absorption with fine spectral resolution greatly exceeds the requirements of radiative transfer calculations. As shown in (2) Chapter 2, it is unfortunately difficult to define averages which permit satisfactory calculations. Thus Planck and Rosseland means which average μ_ν and μ_ν^{-1} respectively apply only in limiting situations; the one for very transparent, the other for very opaque media. Nevertheless such means are useful and have been calculated. In the specific case of line effects, mentioned in the preceding paragraph, the contribution is not very large until one reaches temperatures like $2 \times 10^{5\,o}K$ and high densities.

Because of the many uncertainties entering the calculation of absorption coefficients one has no systematic way of estimating their accuracy. The responsible authors of opacity calculations are generally confident that their results lie within a factor of three of the true values.

In the intermediate temperature range where the opacity reaches a maximum,the accuracy is probably somewhat better. Because of the large opacity the averaging procedures, which are appropriate for radiative transfer calculations, put the most weight on those parts of the spectrum where μ_ν is small and very little weight on the lines. The Rosseland mean does and the Planck mean does not fall into this class. Because of

the emphasis on the continuum, where one has more reliable information, the Rosseland mean is expected to be more accurate.

The extent to which opacity errors falsify fireball calculations depends on the temperature range. Inspecting the temperature profiles of Fig. 2-2 which are typical for the early stage of a fireball one finds large temperatures near the center which makes the air in that region very transparent. Further out the temperature drops so that the opacity rises, goes through a maximum and then drops again. In the transparent central region radiative heat transfer is rapid and keeps that region at a fairly uniform temperature, as one sees in Fig. 2-2. Just how uniform this profile is has very little effect on the rate of expansion and therefore opacity errors by a factor twice or even more are not serious in that region.

The opaque zone around the central region acts as a radiative barrier and the development of the fireball does depend quite critically on the opacity there. During the very early phase where hydrodynamic motion is still negligible compared to the radiative expansion the section of the opacity-temperature relation near the maximum determines the rate of that expansion. It also determines when and where the hydrodynamic shock begins to form.

When shock temperatures are still high, the opaque shell forms in a temperature toe ahead of the shock. This is the precursor which has been sketched in Fig. 2-7 and which causes the early structure in Fig. 2-1. At this stage the opacity is still of interest, since it determines the character of the escaping radiation and other observable phenomena, but the rate at which the fireball expands is given by the shock speed which does not depend on the opacity in the toe.

The next phase starts when the shock temperature has dropped low enough that the shocked air becomes opaque. This is aided by the high density directly behind the shock which can be seen for example, in the density profiles of Fig. 2-3. Up to that time radiative transfer plays a major role in feeding energy to the expanding shock front. Now that source fades out and hydrodynamics takes over as the dominant mechanism for energy transfer. The details of the change depend quite critically on the opacity relation.

Upon further cooling the shock front becomes transparent again and the opaque shell recedes toward the center. This starts the long time interval during which the power vs. time curve of Fig. 2-1 goes through its minimum, rises back to the final maximum, and starts to drop again. The calculation of this phase also depends quite critically on the opacity. A test calculation made with an opacity twice the accepted value stretched the total duration of this phase by almost a factor of two with a corresponding reduction of the maximum power level to about half of what it was in the earlier calculation. Thus, errors in the opacity relation could lead to fairly severe discrepancies between fireball models at the time of the second maximum.

3.3 Radiation hydrodynamics codes

The purely hydrodynamic part of any code is probably as accurate as the equation of state that is being used except for the smearing out of the shock front introduced by the artificial viscosity method. The accuracy of radiative transfer calculations is less certain, unless one is justified in using the diffusion approximation. In that case the limiting factor is

probably the accuracy of the opacity. Difficulties do arise, however, at the front of an opaque shell. Consider, for example, two zones labeled a and b whose temperatures place them on the rising branch of the opacity curve as indicated in Fig. 3-1. As the heating wave progresses these points move up on the curve. The radiation escaping to the outside comes at first from zone a and passes without attenuation through zone b . As zone a climbs higher the radiation rises as T^4 but when zone b becomes sufficiently opaque to take over the role of zone a the power output drops. This cycle is repeated when zone c and others after it climb into the position originally held by zone a . The result of this is a sequence of maxima and minima in the power versus time curve which has no physical reality. This spurious effect can be counteracted by using finer zone sizes but at the expense of increasing the running time which increases as the square of the number of subdivisions per zone. Actually this is not necessary, since test calculations show that the cruder zone divisions lead to the same average power and to the same rate of expansion as a very fine division. A related problem arises when the artificial viscosity routine introduces improper heating ahead of the shock front. Letting the point a in Fig. 3-1 represent the shocked zone and b and c the zones just ahead of the shock, this heating would make points b and c lie at too high temperatures. The calculated attenuation of the radiation from the shock is therefore larger than what it should really be and leads us to predict too low a brightness of the fireball. The reduced output has, however, practically no effect on the calculated motion of the fireball air because at that stage the amount of energy lost by radiation is still too small to influence the hydrodynamics.

Other errors may be introduced by the use of approximate integration routines such as the multiple ray or the two stream techniques which have been discussed in Chapter 1. Given a set of experimental data one can, within limits, adjust the opacity temperature relation so that either model will reproduce these data. It is therefore not really possible to disentangle errors which may arise from the use of these models on the one hand, and from incorrect absorption coefficients on the other hand.

3.4 Deviations from LTE

As we have repeatedly stated, nearly all calculations assume the air to be always in LTE. There are, however, some equilibration processes which are decidedly slow on the time scale of nuclear fireballs. At somewhat elevated altitudes one finds for example, that the processes responsible for populating the vibrationally excited levels of O_2 and N_2 fall into this class. These processes are discussed under the heading "vibrational relaxation" in (4) section 6.1 In the case of O_2 , populating the vibrational levels reduces the photon energy required for reaching the Schumann-Runge continuum below the 8.5 eV which it takes from the ground state. As long as these levels are not populated the actual absorption is therefore less than it would be in equilibrium. Similar considerations apply to the Birge-Hopfield transitions in N_2 . In most codes these delays are just ignored. Hillendahl (see Appendix A) has attempted to account for them by means of a fairly crude model assumption.

Other deviations from LTE are caused by the slowness of chemical reactions at temperatures, say, below $6000^{o}K$ (see (4) sections 6.9 and 7.5.) Among the molecules which can form in this temperature range is

NO_2 which has a large absorption coefficient. The delay in forming this molecule when air is suddenly heated by a shock and the subsequent delay in removing it again when the air cools down can change the absorption significantly from the value at equilibrium conditions. If the temperature drop is rapid enough the NO_2 concentration may stay for a long time at the high concentration corresponding to $3000^{\circ}K$ even though the temperature has dropped below $2000^{\circ}K$. In this situation one speaks of NO_2 as being frozen in.

Non-equilibrium processes also occur at the debris air interface. It has been pointed out in (4) section 5.2 that this is very poorly understood. It is, in particular, quite uncertain what temperature the shocked air would reach and what X-ray spectrum would be emitted by that air.

The questions raised in this chapter clearly do not exhaust the subject of possible errors in the present state of the art. It is, in fact, quite likely that effects with more practical significance have been overlooked. Still, this enumeration should provide the reader with some guidance what he should watch.

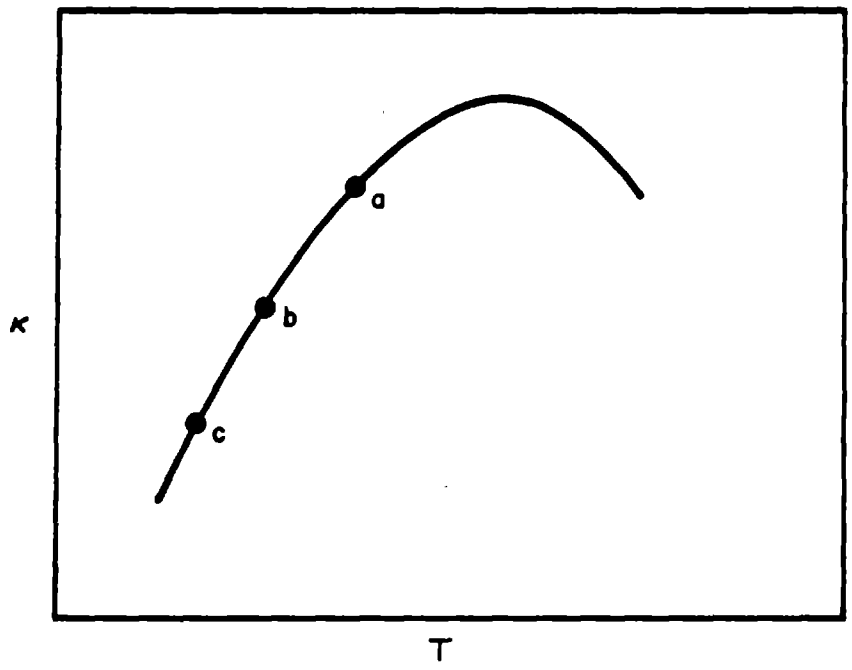

FIG. 3-1 LOCATION OF 3 ADJACENT ZONES ON OPACITY CURVE

Appendix A. A Radiation-Hydrodynamics Code

A.1 Introduction

In this appendix, a sample radiation-hydrodynamics code is presented which employs, with varying degrees of sophistication, much of the physics and basic data presented elsewhere in this volume.

In keeping with Chapter 2, this code describes the radiative and hydrodynamic properties of a sphere of hot air. Details of the weapon itself are not of interest in the present context, and rather crude generalizations have been used to represent the gross properties of the hardware.

The code is presented as a means of demonstrating some of the techniques of radiation-hydrodynamics, as described in Chapter 1, the application of basic physical data, as described in Volume 2, and as an illustration of the results discussed in Chapter 2. The code is not intended as a demonstration of the programming art and has not been polished-up for presentation here. A great deal of the program could be deleted were the program to be used only for present purposes. Much of the basic philosophy of this code has been presented in Chapter 1 and by Hillendahl (1964), and will not be repeated in detail.

The basic equations of the problem are the conservation equations of radiation-hydrodynamics for a one-dimensional spherical system which can be written in Lagrangian form as

$$\frac{\partial U}{\partial t} = -4\pi R^2 \frac{\partial (P + Q)}{\partial m} \qquad \text{Conservation of momentum} \qquad (A.1)$$

$$U = \frac{\partial R}{\partial t} \qquad \text{Definition of velocity} \qquad \text{(A.2)}$$

$$V = 4\pi R^2 \frac{\partial R}{\partial m} \qquad \text{Conservation of mass} \qquad \text{(A.3)}$$

$$\left.\begin{aligned} Q &= c\frac{\left(\rho_o \Delta R\right)^2}{V}\left[\frac{\partial V}{\partial t}\right]^2 \text{ on compression} \\ Q &= 0 \quad \text{otherwise} \end{aligned}\right\} \quad \begin{matrix}\text{Definition of Artificial} \\ \text{Viscosity}\end{matrix} \qquad \text{(A.4)}$$

$$\frac{\partial E}{\partial t} + (P + Q)\frac{\partial V}{\partial t} + 4\pi \frac{\partial (R^2 \mathcal{J})}{\partial m} = 0 \qquad \text{Conservation of energy} \qquad \text{(A.5)}$$

$$E = E(V, T) \qquad \text{(A.6)}$$

$$P = P(V, T) \qquad \text{(A.7)}$$

$\mathcal{J}$ is an integral functional of V and T (A.8)

where

U = local fluid velocity

t = time

R = radius

P = pressure

Q = artificial viscosity

m = mass

c = an arbitrary constant near unity

ρ_o = initial density

V = specific volume (reciprocal density)

$\mathcal{J}$ = radiative net flux at R

E = internal energy

Quite generally, the mathematical formulation of the problem may be characterized as an initial value problem whose solution consists of a time-wise and mass-wise integration of a well defined set of hyperbolic partial differential and partial integro-differential equations.

The solution of these equations is carried out by numerical techniques in which values of the dependent variables are determined in terms of the two independent variables (the time and lagrangian zone mass) by means of finite difference equations which are used to represent Eqs. (A.1) - (A.8).

For purposes of numerical computation, the fireball configuration is represented by a series of concentric, contiguous, spherical mass shells. The mass of the k^{th} zone is designated by m_k (gm/cm^3). Since the mass zones retain their identity throughout the time-wise development of the configuration, the zone index k and the time t (seconds) are convenient choices for the independent variables.

Integration of the set of 8 equations (Eqs. (A.1) - (A.8)) then determines the values of the 8 dependent variables as functions of k and t. U(k,t) (cm sec^{-1}) and R(k,t) (cm) are used to specify the instantaneous values of the interface velocity and radius of the outer surface of the k^{th} mass shell. $\mathcal{J}$(k,t) (ergs cm^{-2} sec^{-1}) is used to specify the instantaneous value of the net radiative flux at the outer boundary of the k^{th} mass zone. P(k,t) (dynes cm^{-2}), Q(k,t) (dynes cm^{-2}), V(k,t) (cm^3 gm^{-1}), T(k,t) (^{o}K), E(k,t) (ergs gm^{-1}) are used to represent the instantaneous values of

the pressure, artificial viscous pressure, specific volume (reciprocal density), temperature and internal energy of the mass zone m_k .

In a purely hydrodynamic problem without radiative transfer, it is the standard practice to reckon the thermodynamic properties of a zone (i.e.: pressure, internal energy, density, temperature) as constant average values over each zone. These values are also considered to be the central values of these variables at the geometrical zone centers. Particle velocities are reckoned at the zone interfaces; the interface density and pressure gradient are formulated in terms of the values at the zone centers. The zoning mesh must be chosen fine enough so that the variation in properties from zone to zone is small enough to insure that these average values are meaningful.

In a problem which also includes radiative transfer, the above restrictions must also be met. Zone sizes in a problem including radiation will generally be smaller than the zone sizes required by hydrodynamics alone. The addition of radiative transfer to the problem will, in general, add further restrictions.

If the temperature is taken as constant across each zone, temperature discontinuities will occur at the zone interfaces. Radiative variables like T^4 will have even greater discontinuities. More detailed examination indicates that the temperature and its spatial derivatives should be continuous at the zone interfaces. Thus, consistent with the expansion used in Eq. (2.5-9), the source function B is taken as linear between zone centers. Then the discontinuous spatial derivatives of the source function which occur at the zone centers do not appear in the formulation. In a more general formulation, a higher order polynomial could be used to

fit the source function through the zone centers, but numerical experience has indicated such a procedure resulted in only minor improvement in the computations.

The central zone temperature T is used as the average over the zone for purposes of computing average zone pressures, internal energies, and is used also in the Z, A, and W computations (Eqs. (2.5-13) through (2.5-15)). This is done primarily for purposes of convenience, but can be at least partially justified. In regions of small temperature gradient, no problem occurs since the central and average zone temperatures are nearly identical. In regions of large temperature gradients, the "average value" of the temperature is poorly defined in terms of the rapidly varying radiative variables, and it is preferable to keep the problem well poised hydrodynamically.

The specific volume V is taken as having a linear variation across each zone. The values of V at the zone centers and zone interfaces are then uniquely defined and afford no further difficulty.

The Z, A, and W functions are then computed using the average zone temperature T and specific volume V. Use of the average specific volume is justified since these functions show a relatively weak density dependence. Neglecting the variations in temperature across the zone causes only relatively small errors in the high temperature inner fireball regions since these functions show only a weak dependence upon temperature. In the low temperature regions, the Z, A, and W functions vary about as the ninth power of the temperature. Even with extremely fine zoning, large temperature gradients occur across each zone, and the Z, A, and W functions would be ill defined in terms

of any average temperature no matter how the average be defined. But the emission from these low temperature regions is small compared to the emission from the high temperature regions further inside, and these layers act primarily as a selective absorber for radiation from larger optical depths. Hence the $Z^{\mp}$ function must be known with some accuracy, but small errors in the $\mathbf{A}$ function, originating because of the use of the average zone temperature, can be tolerated.

The Z^{+} function, however, has peculiar properties in the low temperature region which allow its values to be obtained with sufficient accuracy. As discussed in Chapter 8, the spectral absorption coefficient varies extremely rapidly with wavelength so that the spectrum is effectively divided at some wavelength into absorbed and transmitted fractions. This transition wavelength, however, depends only weakly on the zone temperature, and hence the average zone temperature will again suffice.

It should always be born in mind that these numerical approximations all improve as the zone size is decreased and can, in principle be made accurate to any desired precision. In practice, however, the fineness of the zone mesh is limited by the cost of computation. Skill is thus required to accomplish a large computational program within a limited budget. One tries to test each situation for sensitivity to zone sizes and achieve a compromise between economy of computation and accurate representation of the physics.

For purposes of carrying out the integration procedure by numerical methods, the basic equations (Eqs. (A.1) - (A.8)) are replaced by a set of centered finite difference equations as follows. The notation and

centering can best be seen by reference to Fig. A-1.

$$\frac{U_k^{n+1/2} - U_k^{n-1/2}}{t^{n+1/2} - t^{n-1/2}} = \frac{-8\pi \left(R_k^n\right)^2 \left(P_{k+1/2}^n + Q_{k+1/2}^{n-1/2} - P_{k-1/2}^n - Q_{k-1/2}^{n-1/2}\right)}{m_k + m_{k+1}} \tag{A.9}$$

$$R_k^{n+1} = R_k^n + U_k^{n+1/2}\left(t^{n+1} - t^n\right) \tag{A.10}$$

$$V_{k-1/2}^{n+1} = \frac{4\pi}{3m_k}\left\{\left(R_k^{n+1}\right)^3 - \left(R_{k-1}^{n+1}\right)^3\right\} \tag{A.11}$$

$$\begin{cases} Q_{k-1/2}^{n+1} = \left(\dfrac{c}{V_{k-1/2}^{n+1} + V_{k-1/2}^n}\right)\left(\dfrac{m_k}{4\pi\ R_k^{n+1\,2}}\right)^2\left(\dfrac{V_{k-1/2}^{n+1} - V_{k-1/2}^n}{t^{n+1} - t^n}\right)^2 \\ \\ Q_{k-1/2}^{n+1} = 0 \quad \text{if} \quad V_{k-1/2}^{n+1} > V_{k-1/2}^n \end{cases} \tag{A.12}$$

$$\frac{E_{k-1/2}^{n+1} - E_{k-1/2}^n}{t^{n+1} - t^n} + \left(\frac{P_{k-1/2}^{n+1} + P_{k-1/2}^n}{2} + Q_{k-1/2}^{n+1/2}\right)\left(\frac{V_{k-1/2}^{n+1} - V_{k-1/2}^n}{t^{n+1} - t^n}\right)$$

$$+ \frac{2\pi}{m_k}\left\{\left(R_k^{n+1}\right)^2 \mathcal{J}_k^{n+1} - \left(R_{k-1}^{n+1}\right)^2 \mathcal{J}_{k-1}^{n+1} + \left(R_k^n\right)^2 \mathcal{J}_k^n - \left(R_{k-1}^n\right)^2 \mathcal{J}_{k-1}^n\right\} = 0 \tag{A.13}$$

$$E_{k-1/2}^{n+1} = E\left(V_{k-1/2}^{n+1},\ T_{k-1/2}^{n+1}\right) \tag{A.14}$$

$$P_{k-1/2}^{n+1} = P\left(V_{k-1/2}^{n+1},\ T_{k-1/2}^{n+1}\right) \tag{A.15}$$

$$\mathcal{J}_k^{n+1} = \mathcal{J}_k\left(V_i^{n+1},\ T_i^{n+1}\right) \quad i = 1, \ldots\ldots\ldots\ L \tag{A.16}$$

Eq. (A.9) expresses the conservation of momentum and is centered over the grid point (n,k).

Eq. (A.10) simply expresses the definition of velocity and is centered over the grid point (n+1/2 ,k).

The conservation of mass, expressed by Eq. (A.11), is centered over the grid point (n+1/2 ,k-1/2, i.e. over $Q_{k-1/2}^{n+1/2}$.

The difference expression (Eq. (A.12)) for the artificial viscosity is centered on the grid location (n+1/2 ,k-1/2) except for the R_k^{n+1} factor. The artificial viscosity is thus correctly centered for use in the energy Eq. (A.13), but lags a half time step behind in the equation of motion (Eq. (A.9).

The energy equation (Eq. (A.13)) is centered over the grid location (n+1/2 ,k-1/2) and constitutes an implicit expression to determine the local temperature $T_{k-1/2}^{n+1}$.

Eqs. (A.14) and (A.15) are equations of condition rather than finite difference equations, and express the equations of state for the fluid at the location (n+1, k-1/2) in terms of $V_{k-1/2}^{n+1}$ and $T_{k-1/2}^{n+1}$.

Eq. (A.16), which is written above only in symbolic form, expresses that the local net flux $\mathcal{J}_k^{n+1}$ is an instantaneous integral functional of the temperatures and densities of all of the L zones in the configuration. The set of Eqs. (A.9) - (A.16) thus constitutes a set of 8L equations in 8L unknowns.

The following set of 11 auxiliary equations are used to evaluate Eq. (A.16):

$$ZO_{k-1/2}^{n+1} = ZO\left(V_{k-1/2}^{n+1},\ T_{k-1/2}^{n+1},\ TR_{k-1/2}^{n+1},\ t^{n+1}\right) \tag{A.17}$$

$$ZI_{k-1/2}^{n+1} = ZI\left(V_{k-1/2}^{n+1},\ T_{k-1/2}^{n+1},\ t^{n+1}\right) \tag{A.18}$$

$$\lambda H_{k-1/2}^{n+1} = \lambda H\left(V_{k-1/2}^{n+1},\ T_{k-1/2}^{n+1},\ t^{n+1}\right) \tag{A.19}$$

$$\Delta\tau_{k-1/2}^{n+1} = \left(R_k^{n+1} - R_k^{n-1}\right) \Big/ \lambda H_{k-1/2}^{n+1} \tag{A.20}$$

$$A_{k-1/2}^{n+1} = \exp\left(-\Delta\tau_{k-1/2}^{n+1}\right) \tag{A.21}$$

$$BC_{k-1/2}^{n+1} = \sigma\left(T_{k-1/2}^{n+1}\right)^4 \tag{A.22}$$

$$S_{k-1/2}^{n+1/2} = \frac{BC_{k-1/2}^{n+1} - BC_{k+1/2}^{n+1}}{\Delta\tau_{k-1/2}^{n+1} + \Delta\tau_{k+1/2}^{n+1}} \tag{A.23}$$

$$W_{k-1/2}^{n+1} = 1 - A_{k-1/2}^{n+1} - A_{k-1/2}^{n+1}\,\Delta\tau_{k-1/2}^{n+1} \tag{A.24}$$

$$BB_k^{n+1} = \frac{\sigma\left[\left(R_{k+1}^{n+1} - R_k^{n+1}\right)\left(T_{k-1/2}^{n+1}\right)^4 + \left(R_k^{n+1} - R_{k-1}^{n+1}\right)\left(T_{k+1/2}^{n+1}\right)^4\right]}{\left(R_{k+1}^{n+1} - R_{k-1}^{n+1}\right)} \tag{A.25}$$

$$FO_k^{n+1} = FO_{k-1}^{n+1}\left(\frac{R_{k-1}^{n+1}}{R_k^{n+1}}\right)^2 Z\varphi_k^{n+1} + FI_k^{n+1}\left[1 - \left(\frac{R_{k-1}^{n+1}}{R_k^{n+1}}\right)^2\right] ZI_k^{n+1} + B_k^{n+1}\left(1 - A_k^{n+1}\right) + 2S_k^{n+1}\,W_k^{n+1} \tag{A.26}$$

$$FI_k^{n+1} = FI_{k+1}^{n+1}\,ZI_{k+1}^{n+1} + BB_k^{n+1}\left(1 - A_{k+1}^{n+1}\right) - 2S_k^{n+1}\,W_{k+1}^{n+1} \tag{A.27}$$

$$\mathcal{J}_k^{n+1} = FO_k^{n+1} - FI_k^{n+1} \tag{A.28}$$

where $\sigma = 5.67 \times 10^{-5}$ is the Stephan-Boltzmann constant, and TR is the temperature used to specify the spectral character of the radiation incident upon the zone from regions of smaller radius.

The analytic expressions for Eqs. (A.18) and (A.19) are not given explicitly in the written text, but are available in the FIREBALL code listing which follows.

In carrying out the numerical integration scheme, it is assumed that the initial values of R, T, V, E, P, U, Q, and $\mathcal{J}$ are known at the initial time t^n for all values of k. The difference equations (Eqs. (A.9) - (A.28)) are then used to determine values for these variables at $t^{n+1} = t^n + \Delta t$ for all values of k. The procedure is repeated as n is increased until the desired period of time has been covered by the integration.

The initial values of Q and U are chosen at t^n rather than $t^{n-1/2}$ in the above procedure, but little error is introduced since the initial values may be adjusted accordingly and the time step Δt may be chosen as very small on the initial cycle.

The actual input model consists of a set of R, U, T, and V values for each zone of the configuration. Initial values of Q, E, P, Q, and $\mathcal{J}$ are then found by use of Eqs. (A.12), (A.14), (A.15), and (A.16) before starting the first time cycle.

The set of Eqs. (A.9) through (A.12) depend only on the localized properties of the fluid and they can be advanced explicitly in space and time, subject to the limitations on the time increments according to the Courant criterion (see Chapter 2).

The set of Eqs. (A.13) through (A.28) must be solved simultaneously for all of the L zones in the configuration because of the linkage between distant zones caused by the radiative flux. Since the advanced values

$V^{n+1}_{k-1/2}$ of the specific volumes are known by advancing Eq. (A.11) explicitly, a set of trial temperatures $T^{n+1}_{i-1/2}$, $i = 1, \ldots L$ are estimated, substituted into the Eqs. (A.13) - (A.28), and a Newton-Raphson iteration scheme is then used to adjust the estimated values of temperature until Eq. (A.13) is satisfied to a predetermined accuracy.

The iteration is carried out by numerical methods. The equation of state derivatives, at fixed V, are carried out by raising and lowering the temperature 2% from its trial value, e.g.:

$$\frac{\Delta E}{\Delta T} = \frac{E(V, 1.02T) - E(V, 0.98T)}{1.02T - 0.98T} \qquad (A.29)$$

The derivatives of the radiative quantities are computed by a ripple zone technique in which the temperature of a single zone is raised 2%, the set of Eqs. (A.17) - (A.28) is evaluated and the desired derivatives formed, and the displaced temperature is then returned to its undisplaced value. This procedure is repeated for each zone in the configuration until all the desired derivatives are available. In practice, radiative derivatives for zones more than 2 zones distant are small, so that only a 5 zone set of derivatives is carried. Neglect of the more distant derivatives does not constitute a neglect of radiative transfer between distant zones; their neglect only influences rate of convergence of the iteration procedure.

For a system of L zones, the iteration procedure then results in L linear algebraic equations in L unknown temperature increments, each equation consisting of a set of terms involving 5 of the unknowns. This array is then solved by direct elimination and back substitution. The temperature increments are then used to adjust the trial values of

temperature until all temperature increments for all zones are simultaneously less than 10% of their respective zone temperatures.

The FIREBALL code is written in a programming language called FORTRAN; the particular vintage is known as FORTRAN II, Version 2. The various types of FORTRAN currently in use differ from each other only in minor details. This particular version was selected primarily because it has been in use for a number of years, and has achieved a measure of stability and reliability not to be found in the more recent efforts of the computer industry.

The FORTRAN language little resembles the machine language coding of a decade ago, and its resemblance to ordinary algebra is so close that the average physicist or engineer can learn to read FORTRAN with a very minimum of effort. This allows one to communicate the solution of a complicated theoretical calculation to his fellow scientists in complete detail and complete scientific honesty.

Section A.2 is devoted to a brief discussion of how to read FORTRAN and is designed for the scientist who is not familiar with this type of language. The remainder of Appendix A is devoted to the scientific aspects of the FIREBALL code and is intended to be independent of the language details. Readers not interested in great detail may thus skip over Section A.2, while those interested in such detail will find this section helpful in reading the code itself which is listed in full. The four digit numbers which appear in parenthesis throughout this appendix, e.g. (0136), refer to serial numbers (line numbers) in the code listing.

A.2 Reading FORTRAN

Variables and Constants

Algebraic variables are represented by symbols as in ordinary algebra and may take on values from about $\pm$ 1.7 x 10^{-38} to $\pm$ 1.7 x 10^{+38} , and zero. Arithmetic is accurate to 8 significant figures.

Variable names must consist of 1 to 6 characters, the first of which must be a letter of the alphabet other than I, J, K, L, M, or N.

Examples: YIELD, X, A47, FI, DIVFAR

Integer Variables

Integers are used in subscripts, counting, indices, and sometimes in exponents. They are represented by symbols of 1 to 6 characters and must begin with I, J, K, L, M, or N.

Examples: N, J63, MCOUNT

Subscripts

Variables may have up to three subscripts, but no superscripts. The subscripts may be positive integers, but not zero.

Examples: X(K+1), Z(J,N), BB(I,J,K)

Arithmetic Operations

The symbols for standard arithmetic operations are little different from those used in ordinary algebra:

X + Y means add X and Y

X - Y means subtract Y from X

X * Y means multiply X by Y

X / Y means divide X by Y

Normal algebraic sign conventions are used. A double asterisk is used for exponentiation, i.e.:

R ** 2 means R^2

R ** BETA means R^{β}

Q ** 0.5 means $Q^{1/2}$

Operations are carried out in the preferential order of first exponentiation, then multiplication and division , and finally addition and subtraction.

Equal Sign

The equal sign has a meaning slightly different from algebraic usage, e.g.:

```
XYIELD = XYIELD + (A ** 2) - B
```

means carry out the algebraic operations to the right of the equal sign and store the result of this computation as the new value of XYIELD . Note that the previous value of XYIELD is first used, then destroyed and replaced.

Parenthesis

Parenthesis may be used to group information, indicate subscripts, or indicate the argument of a function.

Examples:

(A+B)*C means add A and B, then multiply by C

X(K+1) means K+1 is a subscript of the variable X

SINF(X) means X is the argument of the sine function

Usually parenthesis will be used for grouping; subscripts can be recognized because they are integers, i.e., I, J, K, etc.; functions can usually be recognized by the letter F and the lack of an arithmetic operation symbol.

Library Functions

Certain library functions may be called in by name in order to save programming labor.

Examples:

EXPF(X) = e^X

SQRTF(Y) = $\sqrt{Y}$

LOGF(Z) = $\log_e Z$

Program Flow

Statements are processed by the computer in order of occurrence unless other directions are provided. Formula numbers, which occur in the first 5 spaces to the left, are not required unless control is to be

switched to that particular formula. The most common control commands are:

GO TO 436	sends control to statement 436
IF(X(K) - 3.0) 450, 500, 750	tells the computer to test the sign of the expression in parenthesis (X-3) and to transfer control to statement 450 if negative, 500 if zero, 750 if positive.

The IF statement provides the only means by which the computer makes judgments.

Subroutines

Complete subprograms, designed to accomplish a particular set of computations many times, can be used. For example:

```
CALL STATE
```

appearing in a program will send the computer to subroutine STATE, the set of computations indicated in that subroutine will be carried out, and control will return to the main program at the statement following CALL STATE.

Repetitive Operations

To save labor in programming, the DO statement may be used:

```
DO 100  K=1, 95
```

instructs the computer to carry out all instructions following this statement until statement 100 is reached. Do this first with K equal to 1 ; repeat this process, increasing K by unity each time, making the final pass with K equal to 95.

This statement can be used to compute repetitive formulae, or it can be used to select values from an array of numbers. For example:

```
      TESTV  =  1.6 E+02          means set TESTV = 1.6 x 10^2
      DO 50  K=1, 100
      IF(X(K) - TESTV)      50,  50,  10
10    XCRIT  =  X(K)              sets XCRIT equal to X(K)
      N  =  K                     retain the value of K as the symbol N
      GO TO 70                    exit from the DO statement
50    CONTINUE                    a dummy statement
      XCRIT  =  0.0               set XCRIT to zero
70    YSTART = XCRIT + DELTAY     the next step in the computation
```

selects the first X value that is greater than 160 out of a list of 100 values of X, starting at X(1),.....X(100). If no such value is found, then XCRIT is set equal to zero.

The above material has been brief and oversimplified, but should enable those unfamiliar with FORTRAN to decipher formulae from a program listing and to follow the scheme of computation. Additional help can usually be obtained from programmers if additional questions of interpretation should arise. Books on FORTRAN are available, but none are recommended.

A.3 The computational scheme

The general computational scheme is cyclic in nature, each completed cycle representing an advance in the time variable.

The method of computation is illustrated in the schematic flow chart, Fig. A-2. Each block in this illustration represents a principal feature of the computation; numbers in the upper left corner of each block refer to a line number to the far-right of each page in the code listing.

The computation begins by reading in an initial configuration from data cards. The configuration consists essentially of the initial density, temperature, velocity, and dimensions of each of the 100 zones in the model. The data cards may represent an initial configuration, or the data might represent the results of a previous computer run on a model that is being done in short segments.

If the entry is a restart rather than an initial start, a dummy subroutine rejust (0693) is provided to make minor changes in program control parameters without the need for recompiling the major subroutines.

After reading in the initial configuration, the data is tested for obvious errors (0367), the equation of state subroutine (0387), the radiative properties subroutine (0389), and finally the flux subroutine (0410) are used to complete the details of the initial configuration. The computation cycle proper is then initiated (0402-0405). The initial time, which has been modified (0404), is now restored to its proper value (0123).

A sequence is then used to determine the number of zones, out of the possible 100, that are to be used at the present stage of the computation. (0124-0138). The number of zones in use is continuously adjusted during the computation to avoid unnecessary computation. When the data is loaded,

zone 100 must have the ambient temperature and density at burst altitude. The code then probes inward to a point where the temperature is twice the ambient value or the particle velocity reaches 1 m per second. The number of zones in use is then arrived at by adding a 6 zone safety factor to the result of the above selection.

The next sequence (0139-0162) selects the times at which the data print-out routine (0833) is called, selects the magnetic tapes to be used for the data, and sets up the necessary parameters to integrate the total energy lost by radiation between two data printouts, and to find the radiant power by differentiation of the energy vs time. The data print-out routine will be described in detail in Section A.4. The data printout routines are one-way streets so far as the main stream of the computation is concerned. Data is siphoned off, but no feed back into the computation proper occurs.

The sequence (0164-0177) provides an emergency data saving mechanism for use in case the computation becomes numerically unstable. In order to provide an economical computation, the time step used must be just below the critical value prescribed by the various criteria which will be described in the hydrodynamic routine (0360). These criteria are not fool-proof, and should the computation become numerically unstable (or should the computer operator err), the already completed computation might be lost unless there were a mechanism for restarting. Once the calculation has become unstable, the data presently in the computer may be invalid. The present sequence writes the fireball configuration on a magnetic tape at the completion of each 50 code cycles. Each time another 50 cycles are completed, the tape is rewound and the next configuration

written. Finally, at the end of the run, the last configuration remains on the tape, is read into the computer (0305), and data cards are punched (0312). Should this data card punching process fail, the tape itself is saved and the cards can then be punched directly. At the time of termination due to any type of error, data cards are then available not more than 50 cycles back. On occasion, an instability may occur just after a configuration has been written in the tape, in which case the computation is lost. But this has a probability of occurrence of about 1 chance in 50.

The next two sequences (0179-0257) select the rezoning subroutine (0179) and the zone splitting subroutine (0225). These sequences will be discussed below in Section A.5 dealing with these subroutines. In principle, one tries to remove fine zoning where it is not needed, and to create fine zoning when the physics of the problem so demands. Use of fine zoning throughout is not feasible because of the increased computational costs and also because the computer can only accommodate a total of 100 zones at one time. If the finest zoning in the problem were used throughout, the entire fireball could not be accommodated. So far as is known, other codes (Brode and Whittaker, private communications, 1965) currently in use are rezoned manually by visual inspection. The present sequences are an attempt to carry out this process automatically during the computation.

The sequence (0259-0264) provides for the printing of detailed diagnostics at the initial cycle, the next cycle following, and at one selected by data card input. The diagnostic routine (1125) is called for this purpose.

A termination sequence is provided (0266-0342) which is used whenever the computation is terminated, except in those cases where the normal course of the computation is interupted by a machine difficulty or an operator error. The termination sequence provides for a final data printout (0274), the printing of program diagnostics (0273), the punching of current data cards (0275-0282) for possible restart purposes, completes, copies, combines, and unloads various tapes (0283-0326), and punches data cards containing the configurations from 1 to 50 cycles prior to the termination (0303-0319), before completing the run (0343).

The series (0123) through (0345) has dealt primarily with control mechanisms, rather than the actual computation, which is resumed at (0346). In the course of the computation it is necessary to carry most of the program variables for all mass zones, but only for the current point in time. A few of the variables must be carried for all space zones, but for two consecutive points in time. For example, the present value of the specific volume for zone K is represented by the symbol V(K), while the specific volume from one time-step in arrears, the retarded value, is represented by VR(K). As the program cycles, the present value becomes the new retarded value, and a new "present" value is computed.

The shifting process, in which the retarded values are set equal to the present values, takes place in the next sequence of commands (0346-0356). Also in this sequence, the present values of the variables are independently saved under the symbol W(K,I) for possible use in the event that the entire cycle requires restarting. The restarting procedure is discussed at the end of this section.

Following retardation of the variables, a hydrodynamics routine (0360) is called, which will be described in detail in Section A.6. This routine advances the velocities, radii, specific volumes, and artificial viscosities.

One of the more important steps in the program is accomplished by a single dummy statement (0364). The first step in solution of the energy equation (Eq. (A.13)), is the estimation of a new trial temperature for each zone. Skillful selection of the trial values will make the iteration process converge faster and thus speed up the computation. However, in an iteration process the equation being solved is never satisfied identically, and any prejudice used in estimating the new temperatures may tend to impose non-random errors in the final results. For this reason, the old values of temperature are used as the first estimates for the new values. The iteration scheme itself then, in a sense, becomes a basic part of the system of equations. As will be discussed later, the iteration scheme used employs the same set of equations as are used to represent the energy equation itself. This procedure should tend to minimize any bias.

The iteration cycle proper (0365) begins by testing the set of temperatures for negative or zero values. Such values are likely to occur in the event of a numerical instability, in which case the computation is immediately terminated through the sequence previously described.

In the next sequence (0371-0387) the equation of state subroutine (1217) is called on three successive occasions. This subroutine is discussed in Section A.7. At this point in the computation, the advanced values of the specific volume are known for each zone. A trial set of temperatures has been selected. The derivatives of the internal energy and pressure,

with respect to temperature, at constant specific volume, are formed by first increasing the temperatures 2% above the trial values, passing through the equation of state to obtain energies and pressures, then repeating the process after lowering the temperatures 2% below the trial values. The required derivatives are then formed numerically from the above data (0384-0385).

The temperatures are then returned to the original trial values and a final pass through the equation of state is made to obtain trial values of the internal energy and pressure (0387).

Two subroutines are then called to provide the necessary radiative flux divergences for use in the energy equation. Subroutine SWABZ (0490) uses the values of V, T, and the dimensions of the zones to compute values of the SWAB and Z functions and is discussed in Section A.8. These values are used in subroutine FLUXS (1183) to compute fluxes and flux divergences as described in Section A.9.

A temperature test occurs next in the computation, but is bypassed on the initial trial of the energy equation.

The energy equation itself is then evaluated (0415-0424). A residue, comprising the imbalance of the energy equation due to the use of trial temperatures, is computed for each zone (0422).

The temperature iteration of the energy equation then commences (0425). Two types of iteration schemes can be selected. If the configuration is entirely optically thin (i.e. the total optical depth from the outer edge to the center is less than a given value) a non-radiative iteration (0429) can be used. Radiative transport is still included in the energy balance, but explicit radiative derivatives do not appear in the iteration scheme. If this alternative is selected, the temperature increments for each zone are

computed immediately (0431).

When radiation is included in the iteration (0439), the radiative and hydrodynamic derivatives are carried only in the inner zones, while only hydrodynamic derivatives are carried in the outer zones.

In the radiative iteration procedure, subroutine COEFF (1395) is called to form the required coefficients for the solution (0447) for the temperature increments. This subroutine is described in Section A.10.

Once the full set of proposed temperature increments is known, they are tested (0457) to see if they are within arbitrary bounds which have been developed by experience. Should these bounds be exceeded, there is a high probability that the time increment used by the computer was too large. Program diagnostics are then printed, the variables are restored to their values at the beginning of the cycle, and the cycle is repeated using a smaller time increment. This recycling is allowed 3 times, after which the computation proceeds even though the test bounds were violated. If the test criteria were correct, a numerical instability results, and the computation is terminated from one of the several points in the program where the instability can be positively detected. An instability does not always occur, however, since the tests are not infallible.

Usually a recycling occurs when a time increment just slightly too large has been used and a single recycle cures the difficulty. Use of this stability check "after the fact" enables the various time step selection criteria to be pushed close to their limits. Use of large safety factors in these criteria, as is the common practice, would be prohibitive as the total computation time would be seriously increased. If the proposed temperature increments satisfy the stability tests, they are accepted and

the trial temperatures are modified accordingly (0485).

These new temperatures become a new set of trial temperatures and the iteration cycle is begun again (0367). The iteration convergence test (0409) is made well into the second and subsequent passes through the iteration sequence. Should the test be satisfied, then the best values of internal energy, pressure, and fluxes, etc., are available as the cycle is completed. If the test is failed, the iteration cycle then progresses with all the necessary data. The code as written performs some unnecessary computations in that the state derivatives are computed but not used if the convergence test is satisfied. This technique was a compromise between several alternatives and was adopted because it required less computer storage at a period when the program was storage limited.

The actual convergence test used (0408) consisted of the requirement that the temperature increments for all zones be simultaneously less than 10% of their respective zone temperatures. This method proved to be more efficient than several other methods tried which were based directly on the degree of imbalance of the energy equation.

The number of passes through the iteration sequence is limited to 3 on any one code cycle (0416). It is a characteristic of the Newton method that convergence takes place very rapidly or not at all, and that on odd number of attempts is usually better than an even number. Should an abnormally large temperature increment occur after 3 iterations, a convergence check at the beginning of subroutine HYDRO detects this and forces a smaller time step on the following cycle. This test differs from the main program stability check in that it is applied only after 3 full iterations and uses stricter criteria.

A.4 Data printout routines

The data printout sequence is contained in subroutine CGSPO (0833), This routine samples the data from the computation proper, generates additional parameters of interest from this information, and produces two magnetic tapes: the main listing and the user tape.

The general problem of determining observables from a list of temperature and density values is a difficult one. Many of the "observable" parameters generated by the printout routine are extremely crude and must be used with extreme caution. These "observables" have been printed below the main tables of data and include the shock radius, fireball radius, effective temperature, color temperature, and the spectral distributions. For example, the code defines the fireball radius (FBR) as the radius at which the optical depth is 0.44 as measured from outside the fireball. This simple definition is easy to code and is useful radius to have printed out. But for comparison with an experimental radius measured photographically, a complete brightness profile is required for comparison with the corresponding densitometer traces from the experiment.

The routine begins by testing for temperature inversions (0864) and causes program diagnostics to be printed if such inversions are found. Placement of this test in the printout routine causes the diagnostics to coincide in time with the data printout so that any unusual features seen in the listings may be studied in detail.

In the sequence (0868-0874) a number of variables to be printed out are set up. This sequence works in conjunction with the sequence (0139-0162) in the main program. The variable CN(LZ) is the average time step between two successive printouts. The variable SAVE1 is the value of the last time step before the printout. The total power being radiated

in watts (POWER), the color temperature (TCOL), and the effective temperature (TEFF) all depend upon the value of the total flux (TFLUX) which is the power being radiated in ergs. TFLUX is computed in the main program (0155). The instantaneous total flux at the end of each cycle is called FLOX and is summed continuously (0139) throughout the computation and called FLEX. The average power TFLUX over the time interval between printouts is obtained by numerical differentiation of FLEX (0155). This method gives more representative results for the power output since the instantaneous power tends to fluctuate due to the finite zoning of the model. The instantaneous total power corresponding to the time of a given printout can easily be obtained since the radius and outward flux at the last zone are available from the printout.

The code definition of the effective temperature (TEFF) is defined (0874) from

$$TFLUX = 4\pi\,(FBR)^2\,\sigma(TEFF)^4$$

where $\sigma = 5.67 \times 10^{-5}$ = Stephan's Constant.

It is the temperature of a black body radiation having the same size as the fireball and which emits the same total power. This temperature is a minimum estimate of the fireball "surface" temperature since the fireball does not have an emissivity of unity.

The code definition of (0873) color temperature (TCOL) uses an estimate of the total emissivity and yields a higher temperature which can be used to describe the spectral character of the radiation escaping from the fireball. The emissivity estimate is based on the Z^+ function

for the zone having the largest λ_c and the radiation temperature for that same zone (0689).

After the fireball becomes transparent in the continuum, the effective and color temperatures are no longer defined since their definitions involve the fireball radius, and by the code's definition, FBR=0 under these circumstances.

At this point (0875), the printout routine calls subroutine PHOTOG (1061). This routine computes an estimate of the photographic brightness as a function of radius. Using an approximate fit (1073) to the absorption coefficient for the "photographic" region of the spectrum, and a Planck function (1083), the emitted intensity is calculated for chord rays which are tangent to the mid-point of each mass zone. Since the spectral band width of the photographic region is unspecified, only relative brightness values are obtained. The brightness scale, however, remains fixed throughout the entire computation. It is clear that only an estimate can be obtained from this computation since the absorption coefficient is not independent of wavelength across the photographic region of the spectrum, and also because the appropriate absorption coefficient data are not available at temperatures above 20 eV. The data is useful, however, in making comparisons with experimental results and in finding the gross brightness variations across the fireball.

The CGSPO routine next makes an estimate of the spectral distribution of the total radiation emitted by the fireball. The code decides (0877) whether the fireball is optically thick or is transparent. If the fireball is transparent (0878), no realistic estimate of the spectral distribution can be made since the spectrum consists of emission lines and a weak

continuum. An extremely crude estimate of the "visible power" (4000-7000 Å) can be made by noting that fireballs tend to form an isothermal region at these late times. Most of the emitted radiation must come from this isothermal region since it is the hottest part of the fireball. These isothermal region temperatures range from 9500°K down to 5000°K as the transparent fireball cools. If it is assumed that the envelope of the emitted spectrum crudely approximates a Planckian distribution, then 37 ± 3% of the radiation will fall in the "visible" region of the spectrum over this entire range of temperatures. Hence, for the transparent case, the visible power (P47) is taken as 37% of the total power (0878).

If the fireball is optically thick (0888), then the color temperature is used as a basis for division of the total power into broad spectral bands (0889-0939). The formula for the Z^+ function, which very nearly approximates the fractional Planck function in the visible and IR spectrum, was used in place of an accurate fit to $\int_{\lambda}^{\infty} B_\lambda \, d\lambda$ as an analytic fit to this integral was not know to be readily available. The accuracy so achieved is probably better than the physics of the spectral estimation process. The overall results are of a quality comparable to those achieved by assuming the sun to be a 6000°K black body.

The next sequence (0944-0963) performs an energy check by summing the internal and kinetic energy of the current configuration. The sequence computes the internal energy and the kinetic energy zone by zone, and also the internal energy of the volume of space currently occupied by the configuration if there had been no detonation. This "ambient" energy of

the undisturbed air constitutes a significant part of the total energy of the configuration, particularly at late times when the radius of the configuration is large. It must be taken into account in studying the energetics of the model.

The code has no built-in method of forcing energy conservation. The energy check sequence is simply an after-the-fact sampling to see if energy has been conserved. If the difference equations do in fact represent the basic conservation equations, then mass, energy, and momentum will automatically be conserved.

The energy accounting at any one moment can be verified by adding the present model energy due to the detonation and the thermal losses up to the given moment. The quantities TOT ES, EAMB and TYIELD are printed out and are the appropriate numbers to use for this purpose. TOT ES in the printout is the total energy, (DPP(LZ) in the code (0959),) and includes EAMB, the energy content of the cold air before the detonation. TYIELD in the printout, (FLEX in the code (0139),) is the total energy radiated outward across the outer boundary of the model from the time of detonation up to the present time. In general, the code conserves energy to within a fraction of 1%.

Sandwiched in the energy check routine are some operations which concern the printing out of shock parameters. (0955-0958). It should be remarked that the location of shock fronts in a numerical configuration is a difficult problem because of the varying number of shocks and the wide range in their characteristics. Many methods were tried, but with limited success. Shocks are best located by a careful study of density vs. radius

and velocity vs. radius graphs.

The location of a strong shock is determined by the program by finding the largest value of the artificial viscous pressure , with the added requirement that it be larger than the ambient gas pressure at burst altitude. This shock radius is printed, but cannot be relied upon without further inspection to see if a shock really exists at that radius. This simple technique will print out the location of one shock, but it may not select the same shock on consecutive printouts.

The sequence (0978-0991) simply changes units and sets up certain quantities in proper form for printing them out. For example, (0987) computes the effective value of "gamma," while (0984) yields the temperature in electron volts.

The sequence (1001-1016) prints out the main listing, while the sequence (1022-1029) prints out the user magnetic tape for use in continuation computations such as fireball environmental studies.

A.5 Variation of the number of zones in use

It has been said that there are three major problems to be solved in writing a fireball code: the basic physics, the interpretation of the results, and the use of proper zoning. Proper zoning is by far the most difficult of these problems. One simply cannot use a fine mesh throughout the configuration as the computing time increases approximately as the square of the number of zones, i.e. halving a zone halves the time step and doubles the number of computer operations, thereby requiring four times as much computer time.

In the other extreme, if the model is represented by zones of equal size, some of the zones may be too large to properly represent the gradients in physical properties.

Consider, for example, a model where the fireball has a radius of 10^3 cm and may require zones of width 10^{-1} cm at the fireball boundary. Since the computer can accommodate only about 100 zones at a time, the zone obviously cannot be of equal size. Furthermore, use of a 10^{-1} cm zone in the hot isothermal region would slow the computation by at least a factor of a hundred, and perhaps a factor of ten thousand or more.

Ideally, new zones should be created and old zones combined in an optimum manner based on the local physical conditions at each point in the configuration. The subroutines SPLIT (0785) and REZONE (0701) are provided for these purposes.

In the split subroutine, the zone indices for the zones exterior to the zone to be split are first shifted outward (0789-0808) to make room for the new zone.

The zone is then split in half mass-wise (0810-0811). The particle velocity at the new interface is taken as the square root of the average of the squares of the particles velocity at the boundaries of the original zone. The temperature of the two new zones are displaced 10% above and below the temperature of the original zone, (0824-0825) and a similar treatment is accorded the internal energies (0826-0827).

This routine must be considered as only a crude beginning and much work is still being done in developing new techniques. The present routine can only be used before the gradients become large, and even then its use should be discouraged.

The calling sequence for the SPLIT subroutine occurs in the main program (0225-0257). Statements (0228 and 0248) prevent a split more often than each third cycle. Statement (0231) requires a split at any time that a single zone represents more than 8% of the radius of the entire configuration.

The real purpose of zone splitting is to provide very fine zones just ahead of an advancing shock front so that the optical properties will not be too severely distorted. The sequence (0232-0244) attempts to achieve this goal. Optical depth 0.7 from the outside is located (0232) and this zone is tested to see if it is being compressed (0235). Then if the particle velocity is greater than 10^5 cm/sec as is characteristic of optically thick shocks, the five zones immediately exterior are scanned and one per cycle can be split, until a certain minimum zone size is reached. Once the shock has started, the splitting takes place 5 zones ahead of the shock until the shock becomes transparent. The minimum zone size has not yet been formulated in general terms, and must be re-programmed in for each separate run. In the program listing (0240) a zone of 200 cm or more can be split, resulting in a minimum zone size of 100 cm. This is about appropriate for a megaton burst at sea level.

As a final insurance against undesirable zone splitting at high altitudes, statement (0244) prevents splitting at temperatures above $6 \times 10^4 {}^{o}K$.

Subroutine REZONE (0701), which combines two existing zones is on a much more sound basis. Conservation of mass in the zone combining process is trivial (0735), but the conservation of energy and momentum are slightly more complicated. The internal energy of the new zone is taken

as a mass weighted average of the internal energies of the two zones being combined (0733). The conservatism of kinetic energy and momentum for a four zone system (0709-0712), which is collapsed into a 3 zone system, results in two equations for the two unknown particle velocities at the inner and outer edges of the combined zone.

The remainder of the subroutine REZONE consists of a shifting of indices for zones exterior to the fusion, and the addition of a new zone 100 at the outer edge of the configuration.

While this routine works well and quite accurately, some skill is required (but not always attained) in deciding when rezoning should take place.

The calling sequence for rezoning is in the main program (0179-0224). On the basis of optical properties (0182), the outer limit of the region to be scanned (the index NZ TS) and an allowable mass ratio for neighboring zones (PMT) are selected (0181-0202). If at least two cycles have passed since the last rezone (0204), zones 9 through NZTS are tested for size compared to the size of the entire configuration (0205), temperature gradient (0206), density gradient (0207), and mass gradient (0208). If these gradients are less than the allowable artitrary limits, then rezoning is allowed so long as the zone is not undergoing significant compression (0209).

A.6 Hydrodynamics routine

The hydrodynamics routine (1261) is patterned after the artificial viscosity method of treating shocks (see Chapter 2). In addition to advancing four of the basic equations, this routine also controls the size of the time increments and performs miscellaneous other functions.

At the beginning of the routine, the previous temperature increment is tested (1273) against the temperature, and if the increment is too large the proposed time step is cut a factor of five (1277). This test serves several purposes depending upon the values of the previous temperature increment that may be stored in the computer at the moment. If a maincycle has been completed satisfactorily, i.e. all temperature increments are less than 10% of their respective temperatures, then no decrease in estimated time step takes place. If, however, a main cycle was completed after three passes of the energy equation, the 10% requirement may not have been satisfied, i.e., the iteration may not have converged. Should this be the case, then the time step is shortened in the hope that a numerical instability can be avoided.

If the main program stability check (0457) is violated, the program returns to the beginning of that master cycle after restoring all data except the temperature increments. Then, since the HYDRO tests are slightly more demanding than are the main program tests, the time step will also be decreased when the main program senses a difficulty.

The equation of motion sequence is carried out in two separate phases which are separated by criteria to choose a new time increment. The particle velocities are first advanced using the previous value of the time step (1289) so that proper time centering of the difference equation (Eq. A.9)) is maintained.

A new time increment is then selected (1298-1335). First a time step 30% larger is suggested (1299). This value is then reduced should the Courant criterion, Eq. (2.5-4), demand that a smaller value be used

(1302-1310). The time may be further reduced by a radiative criterion (1314-1326). Gross checks are then imposed, as a safety factor, which demand that the new time step can never exceed the previous one by more than a factor of two, or be less than a given minimum value (DTMIN). The value of DTMIN is continually increased during the computation and is kept a factor of 50 smaller than the largest time step that has been used. Should the speed of computation decrease more than this factor of 50, too large a time step (the value of DTMIN) is forced into use and the computation may become unstable and turn itself off. This feature prevents the waste of computer time should the time step conditions become abnormally critical at some point in the configuration.

Superimposed on these criteria is the mechanism for causing data to be printed out exactly at fixed predetermined times (1333-1335). This criterion may further decrease the time step.

After a new time step has been decided upon, new values of the particle velocity are determined by linear interpolation (or extrapolation) so that proper time centering of the equation of motion is maintained (1337).

The radii (1344) and specific volumes (1362) are then advanced in a straightforward manner. The new radii are tested before adoption (1348) to prevent sudden zone collapse should the estimated hydrodynamic time step be too large. A local recycle with decreased time step is then instituted (1358).

It is perhaps appropriate to again mention that the use of large safety factors in the time step criteria result in the use of large amounts of computer time. Considerable economy is achieved by lowering the safety factors, testing the proposed results before closing a cycle,

and re-cycling when necessary. Using this technique and with a little experience, the code can be kept running at near optimum speed.

The sequence (1372-1385) advances the artificial viscous pressure. A variety of formulae are available in the literature for this purpose. The formulae used here is similar to that given by Richtmyer (see Chapter 8 references). This form appears to give better results when shocks reflect at the center of the sphere than do the "linear" and "quadratic" forms. In the final analysis, the use of an artificial viscosity is an art and an adjustable constant (1374) is available to achieve optimum results for any specific situation. Use of too small a constant causes numerical ripples behind the shock, while use of too large a value spreads the shock-over too large a distance and lowers the shock velocity. According to Richtmyer, choice of the arbitrary constant in a manner so as to spread the shock discontinuity over 3-4 zones results in shock velocities and pressures that agree well with laboratory experiments.

The use of an artificial viscosity causes a ficticious precursor ahead of the shock front. If this precursor is optically thick, the shock radiation rate will be affected. This effect reduces the rate of radiation loss when the shock temperature is large, but numerical testing has shown that it has little effect on the shock energetics.

A.7 Equation of state routine

The equation of state subroutine (1217) is entered with known values of the temperatures and specific volumes for each zone, and values of pressure and interval energy are computed.

The source data for the analytic fit to equation of state are primarily due to Gilmore (1955) and to Hilsenrath and Beckett (1955). This polynomial type of fit, though in principle not as accurate as an iterative routine, is preferred for computational purposes because the derivatives are well behaved. This fit is also self consistent in that the hugoniots are closely satisfied, while some of the piece-wise fits, that are accurate over limited ranges, fail in this regard. No significant inaccuracies are known to have resulted from use of this simple expression.

There is some question as to whether radiation pressure and the radiation energy density should be included in the equations of state (see Chapter 2). The subroutine **STATE** allows an explicit choice to be made in this regard (1222-3). These effects can only be important at high temperatures. The argument against including these effects is that at the high temperatures the matter and the radiation cannot be in equilibrium according to the local high temperature since the gas is too transparent. The radiation field is characteristic of the lower temperatures where the opacity is higher. To base the radiation pressure on the higher values of temperatures in the interior would thus be an overestimate.

A.8 Radiative properties routine

The radiative properties of the fluid, which are characterized by the S, W, **A**, and **Z** functions, Eqs. (A.17) through (A.25), are supplied by subroutine **SWABZ** (0492-0692). The analytic representations of the $\mathbf{Z}^-$ and **A** functions, which differ only in that the electron scattering is omitted in the **A** function, are formulated in terms of λ_H, a radiative mean free path which depends upon the zone temperature, zone specific volume, and the time.

The Z^- function (ZI in the code) is formulated directly as a transmission (0500-0507).* The A function cannot be formulated directly in this fashion since ln A is needed to define optical distance, and truncation errors would occur as $A \rightarrow 1.0$. Thus λ_H, (HMFP in the code, is found (0508-0511), together with a separate high temperature emission term (0512) due to Bremsstrahlung (HMFF in the code). The smaller of these two mean free paths is used for temperatures above 10 eV (0514).

The emission optical depth (0516) then follows immediately from the zone thickness. The A function (0517) at this point in the code is identical with the definitions used in Eqs. (A.21) and (2.5-14). A later sequence (0670-0675) redefines A as (1-A) (an emissivity) as a computational convenience for zones of small emissivity.

The variables BC and BB 91518 and 0526) are used to represent the source function at the zone centers and zone boundaries respectively. It will be noted that the boundary value is determined by linear interpolation of the source function in terms of geometrical rather than optical distance. The two methods yield substantially the same results when the two neighboring zones have comparable optical thicknesses, but the geometrical method gives better values when an optically thick zone occurs next to an optically thin zone. The terms involving the boundary values cancel if both zones are optically thick, while the source function gradient is small for two transparent zones, so that the method of interpolation is not important in these cases.

The sequence (0536-0563) computes the optical depth inward from

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* The double subscripting, i.e. A(K,N) has been carried over from an earlier version of the code in which N spectral bands were used.

the outer boundary of the configuration and sets various control variables used by the program at other locations. The index LZR specifies the last zone to be included in the radiative part of the iteration scheme. The fireball radius, FBR, is set as the outer radius of the zone in which the total optical depth as measured from the outside has reached 0.44. Choice of this value has no particular significance. For high air densities the optical depth increases abruptly so that any reasonable test value will result in choosing of the same zone to specify the radius. For rarefied atmospheres, considerable limb darkening takes place and chord integrations would be necessary to find a precise radius. The test value of 0.44 has been found to yield radii satisfactory for the intended applications elsewhere in the code. The index MCP is used to specify the zone in which optical depth unity occurs as measured along the representative ray. (This corresponds to a radial optical depth of 2/3).

The sequence (0564-0607) computes the wavelength at which the principal spectral absorption edge occurs for each zone. This wavelength depends only upon the physical characteristics of each zone, i.e., its temperature, density, and zone thickness. Zones are first sorted according to temperature to determine whether the spectral transition is due to an atomic species, the nitrogen molecule, or the oxygen molecule.

The transition edges due to atomic species are in the far ultraviolet and a fit will be required only at very high altitudes when the fireball is transparent throughout while the temperatures are high enough so that the Planck function is significant in the ultraviolet. This fit used by the code is therefore very limited in its application.

The nitrogen and oxygen molecular formulations involve a continuum with a relatively sharp long wavelength limit, and an extensive system of molecular bands extending from this limit toward longer wavelengths. The continua are assumed to exist at all times that the molecules have sufficient population, but the band systems, which depend upon the population of the higher vibrational levels, are assumed not to exist until the passage of a vibrational relaxation period.

While in principle the relaxation time should be measured from the time that each zone is first heated (actually the temperature-time history should be taken into account), a satisfactory first approximation is given by use of the time since the detonation. The relaxation time for N_2 at sea level is taken as 5×10^{-7} sec and for O_2 as 3×10^{-7} secs. These times are scaled inversely with the density to obtain the relaxation times at higher altitudes. These times are based on data by Blackman (1956).

The absorption edge and the continuum absorption due to a particular species must fade away as the population is diminished by dissociation. At a given density, the population decreases very rapidly with temperature as soon as kT becomes comparable with the dissociation energy. Due to the finite zoning structure, the population of a particular specie will be appreciable in one zone, but negligible in a neighboring hotter zone due to the rapid temperature dependence of the populations. Thus, for the purpose of computing absorption edges, the species can be assumed to exist below a certain temperature and not to exist above that temperature. The dissociation is thus assumed to take place at a temperature rather than over a narrow temperature range. The dissociation temperatures for N_2

and O_2 , as a function of density, are represented in the code by two formulae (1708-1709) and are used in the SWABZ subroutine (0572-0580).

The sequence (0612-0669) computes the Z^+ values (ZO in the code). The Z^+ function depends on λ_c which has been determined above, and the temperature T_R used to describe the spectral distribution of the radiation incident upon the zone. The sequence (0622-0642) selects this temperature for each zone. In the case of an optically thick fireball, this temperature is that of the first zone at optical depth greater than 0.7 from the zone in question (0628). For a transparent fireball, where the radiation comes from a shell rather than from a "surface", the temperature is taken as an optical depth weighted average of the zones interior to the zone in question (0624). This "shell source" sequence is important only at high altitudes where the fireball is transparent, and when the temperatures are still high enough to have radiative flow in the far UV region of the spectrum where the absorption edges occur.

The Z^+ values are corrected for the effects of intervening zones between the source of the radiation and the zone in question (0655).

The sequence (0676-0688) becomes effective at very high altitudes when the configuration is quite thin and thermal radiation plays only a minor part in determining the temperature distribution. Under these circumstances, the series expansion of the source function should not be truncated and derivatives of higher order than the first should be included. But since the radiation is of little importance in this case, the series can be further truncated so that only the zero order term is used and the code automatically switches over to radiative transfer for isothermal optically thin slabs.

The variable EMS (0689) is used as an estimate of the total emissivity. It corresponds to the Z^+ function for that zone having the maximum cut-off wavelength, but without correction for intermediate zones.

A.9 Radiative flux routine

Since all the needed data has been generated elsewhere by the code, the radiative flux routine is particularly simple (1183).

To perform the flux integration a boundary condition is needed. For this purpose, the inward flux incident on the outer boundary of the configuration is chosen as zero (1190).

The inward flux at the boundary, one zone inside, is given by the sum of the transmitted and emitted components (1193) as in Eq. (A.27). The integration continues inward until the spherical central zone is reached. The outward flux for this zone is the sum of the transmitted inward flux and the local emission (1195). The integration then proceeds outward (1197) using Eq. (A.26). One can view this process as an integration starting at one side of the fireball, passing through the center, and then on out the other side. All of the inward and outward fluxes needed to form the radial component of the flux divergences (1211) are generated during this integration.

It should be noted that this subroutine is written again for N spectral bands, and that the total directional flux at a given boundary is obtained by a summation over the spectrum, even though only 1 spectral band is used by the present code. In addition, the A function used in (1193-1198) does not correspond to the transmission function used elsewhere in the text, but to (1-A) as explained in the SWABZ subroutine.

The energy Equation (Eq. (A.5)) is solved by an iterative method as described by Hillendahl (1964). Subroutine COEFF (1395) performs the necessary algebra to form the required derivatives.

The basic approach is to compute the derivatives numerically by a ripple zone technique, rather than to derive and to code explicit formulae for the derivatives. In using this technique, the temperature of a single zone is increased 2%, then formulae identical to those used in subroutine SWABZ are used to calculate those radiative properties which change as a result of this temperature increment (1429-1576). A flux integration is then carried out (1577-1593), the flux derivatives are formed (1614), and the perturbed variables are returned to their normal values (1627) which were saved (1411). This ripple zone process is repeated until all the required derivatives are available. Derivatives with respect to zone temperature more than two zones distant are truncated.

The energy equation derivatives, including both the radiative and hydrodynamic parts, are formed numerically (1647). The data is then available to form a set of linear algebraic equations for the temperature increments of the zones.

This matrix is then solved by direct elimination and back substitution using recursion formulae (1673) and (0447).

This numerical method of solution, involving the ripple zone method of obtaining flux derivatives and the step by step formation of the energy equation derivatives has been found to be both convenient and economical. If details of the radiative properties or flux formulae are changed, duplicates of the new formulae are simply inserted into the coefficient subroutine without the necessity of deriving explicit formulae for the

derivatives, which in some cases, have as many as 40 terms. Such lengthy formulae are difficult to derive and code without errors, and the sorting of terms to avoid arithmetic truncation difficulties is a major task. Since the actual number of operations to be carried out by the computer is approximately equivalent in the two methods, the numerical technique is to be preferred.

A.10 Code listing

The pages following present a complete listing of an actual working radiation-hydrodynamic code which has been described in sections A.1-A.9. No attempt has been made to edit the listing for publication purposes since this practice quite often results in the publication of codes that do not work.

```
*     FORTRAN                                                                0001
CFIRE 1                                                                      0002
C    MODIFIED DIMENSION STATEMENT   NSZ=1  ONLY                              0003
      DIMENSION    T(101),   TS(100),  TP(100),  TM(100),  TR(100),         0004
     2   PA(100),  P(100),   PS(100),  PP(100),  PM(100),  PR(100),         0005
     3   DE(100),  E(100),   GM(100),  EP(100),  EM(100),  ER(100),         0006
     4   Q(100),   V(100),   VS(100),  RHOZ(100),U(100),   VR(100),         0007
     5DR(100),D(1),R(101),   RZ(100),  R2(100),  UA(100),  UR(100),         0008
     6   DTM(100), DTR(100), DTH(100), DPT(100), DET(100), RR(100),         0009
     7   ZMAS(100),DIVFA(100),DIVFR(100),RESDUE(100),FP(100), FZ(100),      0010
     8   FM(100),  FMM(100), HZ(100),  HP(100),  HPPP(100),HPP(100),        0011
     9   DMM(100), DM(100),  DZ(100),  DP(100),  DPP(100), BN(100),         0012
     1 CN(100),DN(100),EN(100),ETA(100),XS(100),GNU(100),PART(100)          0013
      DIMENSION THETA(100)                                                   0014
C                                                                            0015
C    STORAGE FOR RADIATION VARIABLES                                         0016
C                                                                            0017
      DIMENSION  BB(100,10),   DTAU(100,10),  S(100,2),    FO(100,10),     0018
     1FOT(100,10),FI(100,10),  FIT(100,10),  ZO(100,10),ZI(100,10)          0019
     2,A(100,10),  W(100,10),   BC(101,10),   FIS(100),  FOS(100),          0020
     3CWL(100, 2),YM(100),TAU(100, 2)                                       0021
      COMMON T,TS,TP,TM,TR,PA,P,PS,PP,PM,PR,DE,E,GM,EP,EM,ER,Q,V,VS,RHO     0022
     2Z,U,VR,DR,D,R,RZ,R2,UA,UR,DTM,DTR,DTH,DPT,DET,RR,ZMAS,DIVFA,DIVFR     0023
     3,RESDUE,FP,FZ,FM,FMM,HZ,HP,HPPP,HPP,DMM,DM,DZ,DP,DPP,BN,CN,DN,EN      0024
     4,ETA,XS,GNU,PART,THETA,KGU,KGP                                        0025
     5,BB,DTAU,S,FO,FOT,FI,FIT,ZO,ZI,A,W,BC,FIS,FOS,CWL,YM,TAU,NSZ,        0026
     6 NZSS,  LZ,LZM1,LZM2,LZP1,LZP2,BR,RJ,RX,NCW,NMC,NTI,FLOX,FLEX,        0027
     7TIME,DT,CS,CR,RS,MCL,MCP,MCW,RM,VD,ZA,NB,NQS,FBR,LZR,DTMIN,NR         0028
     8,KZ1,KZ2,KZ3,KZ4,KZ5,KZ6,KZ7,KZ8,KZ,TD02,TDN2,NDP,SCALE,EMS,         0029
     9BLANK,AST,TEE,PLUS,PERIOD,DASH,EQUAL,PINUS,FFF,UUU,PPP,NTAPE,TIMEW   0030
     6,TFLUX,FLIX,TIMES,P03,P34,P45,P57,P71,P47,Q47,Q71,WKT,YIELD,XYIELD    0031
      KZ6=0                                                                 0032
      REWIND 41                                                             0033
      REWIND 31                                                             0034
      REWIND 32                                                             0035
      REWIND 22                                                             0036
      REWIND 42                                                             0037
      REWIND 25                                                             0038
B     BLANK=606060606060                                                     0039
B     AST=545454545454                                                       0040
B     PLUS=202020202020                                                      0041
B     PERIOD=333333333333                                                    0042
      D(1)=0.0                                                              0043
      TIMEW=1.0E-10                                                         0044
                                                                            0045
40    FORMAT(17HCONTROL DATA RUN 9I4,6X,E10.3,11A1)                          0046
41    FORMAT(14HCONSTANTS RUN I2,7H CYCLE I4,1X,1P3E12.5,0P4F4.2)            0047
42    FORMAT(17HDATA CARDS CYCLE I4,10X,10HRUN NUMBERI3,10X,5HSET A/         0048
42   1(I4,1P5E12.5,1PE11.4,I2,I3))                                           0049
43    FORMAT(17HDATA CARDS CYCLE I4,10X,10HRUN NUMBERI3,10X,5HSET B/         0050
43   1(I4,1P5E12.5,1PE11.4,I2,I3))                                           0051
```

```
44    FORMAT(1H ,6H MODE=I4/6H  NMC=I4/6HNZONE=I4/6H  NQS=I4/6H  MCL=I4/   0052
44   16H   RS=1PE11.4/6H TIME=E11.4/6H   DT=E11.4/6H   CS=E11.4/6H   CR=   0053
44   2E11.4/6H   BR=E11.4/7H TIMEL=1PE11.4)                                 0054
45    FORMAT(1H1,2I4,1PE12.3,I5,1P6E12.3/(I4,1P12E10.3))                    0055
47    FORMAT(1H ,18HZONE SPLIT AT NMC=I4,5X,5HTIME=1PE11.4,5X,5HZONE=I4,    0056
47   119)                                                                   0057
48    FORMAT(1H1,30X,4HRUN=I4,15X,20HLOG POWER (ERGS/SEC)/6H CYCLE,X,4HT    0058
48   2IME,3X,I2,13X,I2,13X,I2,13X,I2,13X,I2,13X,I2,X,11HTOTAL POWER,3X,    0059
48   36HRADIUS/15X,76A1)                                                    0060
49    FORMAT(1H ,15HREZONED AT NMC=I4,5X,5HTIME=1PE11.4,5X,5HZONE=I4,I7)    0061
50    FORMAT(I4,1P5E13.5)                                                   0062
      NSZ=1                                                                 0063
      REWIND 15                                                             0064
      REWIND 16                                                             0065
      READ INPUT TAPE 5,40,NR,MODE,NMC,NLP,NDD,LZ,NZONE,NQS,MCL,TIMEL       0066
     1,BLANK,AST,TEE,PLUS,PERIOD,DASH,EQUAL,PINUS,FFF,UUU,PPP               0067
      PRINT 8321, MCL,TIMEL                                                 0068
8321  FORMAT(34H NORMAL TERMINATION CONDITION NMC=I4,10X,5HTIME=1PE12.3)     0069
      WRITE OUTPUT TAPE 15,12345,NR                                         0070
12345 FORMAT(1H1////////////1H ,40X,  32HR W HILLENDAHL PALO ALTO 201        0071
123451//1H ,40X,33HPRODUCTION OUTPUT LIST RUN NUMBERI4)                     0072
      READ INPUT TAPE 5,41,NR,NMC,TIME,DT,FLEX,CS,CR,BR,RS                  0073
      NSTART=NMC                                                            0074
      DTMIN=DT                                                              0075
      NEG=NMC                                                               0076
      MCOC=NMC+50                                                           0077
      NMA6=MCOC                                                             0078
      JXC=0                                                                 0079
      NMCS=NMC                                                              0080
      TIMES=TIME                                                            0081
      FLIX=FLEX                                                             0082
      NSTOP=0                                                               0083
      NRZ=50                                                                0084
      NB=14                                                                 0085
      IPS=0                                                                 0086
      LZM1=LZ-1                                                             0087
      LZM2=LZ-2                                                             0088
      IF(MODE-1)1,1,1000                                                    0089
1     CALL ENTRY                                                            0090
      WRITE OUTPUT TAPE 6,44  ,MODE,NMC,NZONE,NQS,MCL,RS,TIME,DT,CS,CR,     0091
     1BR,TIMEL                                                              0092
      FBR=R(100)                                                            0093
      RTEST=R(1)                                                            0094
      NPL=NMC                                                               0095
      GO TO 1036                                                            0096
1000  READ INPUT TAPE 5,42,NMC,NR,(NMC,R(K),U(K),V(K),Q(K),T(K),P(K),NR,    0097
1000 1K,K=1,100)                                                            0098
      READ INPUT TAPE 5,43,NMC,NR,(NMC,RZ(K),R2(K),ZMAS(K),DR(K),RHOZ(K)    0099
     1,TR(K),NR,K,K=1,100)                                                  0100
      READ INPUT TAPE 5,50,NR,TIME,TEE,WKT,TDN2,TDO2                        0101
      READ INPUT TAPE 5,50,NR,SCALE,EQUAL,Q71,Q47,TIMEW                     0102
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```
      NB=NMC+1                                                          0103
      LZ=100                                                            0104
      NLP=NLP+25                                                        0105
      DTMIN=DT*0.1                                                      0106
      CALL REJUST                                                       0107
      IPS=0                                                             0108
      NPL=NMC                                                           0109
 1034 WRITE OUTPUT TAPE 6,1035,DT,DR(1),(K,R(K),RHOZ(K),V(K),T(K),U(K),Q 0110
     1(K),ZMAS(K),DR(K),K=1,100)                                        0111
 1035 FORMAT(1H1,10HINPUT DATA,10X,3HDT=E12.4,5X,3HDR=E12.4/3H  K,2X,4HR 0112
     1(L),8X,7HRHOZ(K),5X,4HV(K),8X,4HT(K),8X,4HU(L),8X,4HQ(K),8X,7HZMAS 0113
     1(K),5X,5HDR(K) /(I4,1P8E12.4))                                    0114
                                                                        0115
      WRITE OUTPUT TAPE 6,44  ,MODE,NMC,NZONE,NQS,MCL,RS,TIME,DT,CS,CR, 0116
     1BR,TIMEL                                                          0117
      FBR=R(100)                                                        0118
      RTEST=R(1)                                                        0119
1036  NTAPE=15                                                          0120
      GO TO 3000                                                        0121
C MASTER CYCLE RE-ENTRY POINT                                           0122
 2000 TIME=TIME+DT                                                      0123
C   ROUTINE TO CHANGE NUMBER OF ZONES IN USE " " LIMIT 100 ZONES        0124
10    DO 14  J=1,100                                                    0125
      K=101-J                                                           0126
      IF(U(K)-1.0E+02) 11,12,12                                         0127
11    IF(T(K)-2.0*T(100)) 14,14,12                                      0128
12    LZ=K+6                                                            0129
13    GO TO 16                                                          0130
14    CONTINUE                                                          0131
15    LZ=100                                                            0132
16    IF(LZ-100) 18,18,17                                               0133
17    LZ=100                                                            0134
18    LZM1=LZ-1                                                         0135
19    LZM2=LZ-2                                                         0136
20    LZP1=LZ+1                                                         0137
21    LZP2=LZ+2                                                         0138
22    FLEX=FLEX+FLOX*DT                                                 0139
23    IF(NMC) 33,24,26                                                  0140
24    NTAPE=15                                                          0141
      CN(LZ)=0.0                                                        0142
      TFLUX=FLOX                                                        0143
      FLEX=0.0                                                          0144
25    GO TO 732                                                         0145
26    IF(NMC- 3)  27,27,29                                              0146
27    NTAPE=6                                                           0147
28    GO TO 32                                                          0148
29    IF(TIME-TIMEW)  33,30,30                                          0149
30    TIMEW=TIMEW*(10.0**(1.0/18.0))                                    0150
      IF(NMC- 4)  31,29,31                                              0151
31    NTAPE=15                                                          0152
      LZP2=N1                                                           0153
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32    CN(LZ)=(TIME-TIMES)/FLOATF(NMC-NMCS)                                  0154
      TFLUX=(FLEX-FLIX)/(TIME-TIMES)                                        0155
732   CALL CGSPB                                                            0156
      NOP=NOP                                                               0157
      FLIX=FLEX                                                             0158
      NMCS=NMC                                                              0159
      TIMES=TIME                                                            0160
      LZP2=LZ+2                                                             0161
33    CONTINUE                                                              0162
                                                                            0163
494   IF(NMC-MCDC)  414,409,409                                             0164
409   MCDC=MCDC+50                                                          0165
      REWIND 16                                                             0166
      MODE=MODE+1                                                           0167
      WRITE OUTPUT TAPE 16,40,NR,MODE,NMC,NLP,NEG,LZ,NZONE,NQS,MCL,         0168
     1TIMEL,BLANK,AST,TEE,PLUS,PERIOD,DASH,EQUAL,PINUS,FFF,UUU,PPP          0169
      WRITE OUTPUT TAPE 16,41,NR,NMC,TIME,DT,FLEX,CS,CR,BR,RS               0170
      WRITE OUTPUT TAPE 16,42,NMC,NR,(NMC,R(K),U(K),V(K),Q(K),T(K),         0171
     1P(K),NR,K,K=1,100)                                                    0172
      WRITE OUTPUT TAPE 16,43,NMC,NR,(NMC,RZ(K),R2(K),ZMAS(K),DR(K),        0173
     1RHOZ(K),TR(K),NR,K,K=1,100)                                           0174
      WRITE OUTPUT TAPE  16,50,NR,TIME,TEE,WKT,TDN2,TDO2                    0175
      WRITE OUTPUT TAPE 16,50,NR,SCALE,EQUAL,Q71,Q47,TIMEW                  0176
      MODE=MODE-1                                                           0177
                                                                            0178
C  REZONE SWITCH                                                            0179
414   RTEST=R(LZ-5)                                                         0180
      NZTS=LZ-11                                                            0181
      IF(MCP-3)  415,415,4150                                               0182
C    4150   ALL OPTICALLY THICK CASES                                       0183
4150  NZTS=MCP-4                                                            0184
      PMT=2.55                                                              0185
4153  IF(NZTS-9)   427,417,417                                              0186
C    415   ALL TRANSPARENT CASES                                           0187
415   IF(T(4)-2.0*TDN2)   4170,4170,416                                     0188
C    416     HIGH ALTITUDE EARLY PHASE                                      0189
416   DO 4161  L=9,LZ                                                       0190
      IF(T(L)-1.5*TDN2)   4160,4161,4161                                    0191
4160  NZTS=L-1                                                              0192
      PMT=1.5                                                               0193
      IF(NZTS-9)   427,417,417                                              0194
4161  CONTINUE                                                              0195
C    4170   LATE TIME TRANSPARENT CASE                                      0196
4170  DO 4140   J=11,LZ                                                     0197
      IF(R(J)-W(1,9))   4140,4141,4141                                      0198
4141  NZTS=J-5                                                              0199
      PMT=2.8                                                               0200
      IF(NZTS-9)   427,417,417                                              0201
4140  CONTINUE                                                              0202
417   DO 426  K=9,NZTS                                                      0203
4013  IF(NMC-NRZ)  427,427,419                                              0204
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419   IF((R(K)-R(K-2))-0.08*RTEST)  422,422,426                          0205
422   IF(ABSF(T(K)-T(K-1))-0.30*T(K))  423,423,426                       0206
423   IF(ABSF(V(K)-V(K-1))-0.7*V(K))  424,424,426                        0207
424   IF(ZMAS(K)+ZMAS(K-1)-PMT*ZMAS(K-2))  425,425,426                   0208
425   IF(Q(K)-0.05*P(K))  9425,9425,426                                  0209
9425  LZP2=K                                                             0210
      NRZ=NMC                                                            0211
      JXC=JXC+1                                                          0212
      WRITE OUTPUT TAPE22,49,NMC,TIME,LZP2 ,JXC                          0213
      WRITE OUTPUT TAPE 6,49,NMC,TIME,LZP2 ,JXC                          0214
      CALL REZONE                                                        0215
      LZ=LZ-1                                                            0216
      LZM2=LZ-2                                                          0217
      LZM1=LZ-1                                                          0218
      LZP1=LZ+1                                                          0219
      LZR=LZR-2                                                          0220
      LZP2=LZ+2                                                          0221
      NRZ=NMC+2                                                          0222
      GO TO 427                                                          0223
426   CONTINUE                                                           0224
C   OPTICAL SPLIT TEST                                                   0225
427   NS=LZ-5                                                            0226
      KS=0                                                               0227
      IF(NMSP-NMC)  428,428,448                                          0228
428   DO 440   J=3,NS                                                    0229
429   K=LZ-J                                                             0230
      IF((R(K)-R(K-1))-0.08*R(LZ))    430,430,442                        0231
430   IF(TAU(K,1)-1.7)  440,440,431                                      0232
431   KS=KS+1                                                            0233
432   IF(KS-1)  448,433,448                                              0234
433   IF(Q(K)-P(K))  448,434,434                                         0235
434   IF(U(K)-1.0E+05)  448,435,435                                      0236
435   KST=K                                                              0237
      KSP=K+5                                                            0238
      DO 436  J=KST,KSP                                                  0239
      IF((R(J)-R(J-1))-200.0)         436,436,437                        0240
436   CONTINUE                                                           0241
      GO TO 448                                                          0242
437   KZ2=J                                                              0243
      IF(T(J)-6.0E+04)  443,448,448                                      0244
440   CONTINUE                                                           0245
441   GO TO 448                                                          0246
442   KZ2=K                                                              0247
443   NMSP=NMC+3                                                         0248
      JXD=JXD+1                                                          0249
444   CALL SPLIT                                                         0250
      WRITE OUTPUT TAPE  6,47,NMC,TIME,KZ2,JXD                           0251
      WRITE OUTPUT TAPE 22,47,NMC,TIME,KZ2,JXD                           0252
      LZ=LZ+1                                                            0253
      LZM1=LZ-1                                                          0254
      LZP1=LZ+1                                                          0255
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      LZP2=LZ+2                                                          0256
      LZM2=LZ-2                                                          0257
                                                                         0258
448   IF(NMC-NSTART)  449,451,449                                        0259
449   IF(NMC-NSTART-1)  450,451,450                                      0260
450   IF(NMC-NOD)  453,451,453                                           0261
451   KZ2=-451                                                           0262
452   CALL DIAGNS                                                        0263
453   NMC=NMC+1                                                          0264
                                                                         0265
C                                                                        0266
      IF(NSTOP)  35,35,36                                                0267
35    IF(TIME-6.0) 34,52,52                                              0268
34    IF(SENSE SWITCH 1)  52,65                                          0269
52    MODE=MODE+1                                                        0270
      NSTOP=1                                                            0271
      MCL=MCL+3000                                                       0272
      CALL DIAGNS                                                        0273
      GO TO 28                                                           0274
36    PUNCH 40,NR,MODE,NMC,NLP,NEG,LZ,NZONE,NQS,MCL,TIMEL                0275
     1,BLANK,AST,TEE,PLUS,PERIOD,DASH,EQUAL,PINUS,FFF,UUU,PPP            0276
      PUNCH 41,NR,NMC,TIME,DT,FLEX,CS,CR,BR,RS                           0277
      PUNCH 42, NMC,NR,(NMC,R(K),U(K),V(K),Q(K),T(K),P(K),NR,K,K=1,100)  0278
      PUNCH 43,NMC,NR,(NMC,RZ(K),R2(K),ZMAS(K),DR(K),RHOZ(K)             0279
     1,TR(K),NR,K,K=1,100)                                               0280
      PUNCH 50,NR,TIME,TEE,WKT,TDN2,TDO2                                 0281
      PUNCH 50,NR,SCALE,EQUAL,Q71,Q47,TIMEW                              0282
      END FILE 15                                                        0283
      END FILE 25                                                        0284
      REWIND 25                                                          0285
      CALL COPY (25,15)                                                  0286
      END FILE 15                                                        0287
      END FILE 22                                                        0288
      REWIND 22                                                          0289
      CALL COPY(22,15)                                                   0290
      END FILE 15                                                        0291
      WRITE OUTPUT TAPE 6,37,JXC                                         0292
37    FORMAT(1H ,13HREZONE CALLED14,6H TIMES)                            0293
      REWIND 16                                                          0294
      NR=9999                                                            0295
      WRITE OUTPUT TAPE 41,1499,NR,KZ6                                   0296
      WRITE TAPE 31,NR,KZ6                                               0297
1499  FORMAT(214)                                                        0298
      WRITE OUTPUT TAPE 32, 45,NR,NMC,TIME,LZ,P(99),T(99),RHOZ(99),POWER 0299
     2,P47,FBR,(K,R(K),U(K),P(K),UR(K),T(K),FOS(K),FIS(K),DM(K),Q(K),    0300
     3FZ(K),E(K),HPPP(K),K=1,LZ)                                         0301
      IF(NMC-NMA6)  1502,1502,1501                                       0302
1501  READ INPUT TAPE   16,40,NR,MODE,NMC,NLP,NEG,LZ,NZONE,NQS,MCL,      0303
     1TIMEL,BLANK,AST,TEE,PLUS,PERIOD,DASH,EQUAL,PINUS,FFF,UUU,PPP       0304
      READ INPUT TAPE   16,41,NR,NMC,TIME,DT,FLEX,CS,CR,BR,RS            0305
      READ INPUT TAPE   16,42,NMC,NR,(NMC,R(K),U(K),V(K),Q(K),T(K),      0306
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```
     1P(K),NR,K,K=1,100)                                                    0307
      READ INPUT TAPE   16,43,NMC,NR,(NMC,RZ(K),R2(K),ZMAS(K),DR(K),        0308
     1RHOZ(K),TR(K),NR,K,K=1,100)                                           0309
      READ INPUT TAPE16,50,NR,TIME,TEE,WKT,TDN2,TDO2                        0310
      READ INPUT TAPE16,50,NR,SCALE,EQUAL,Q71,Q47,TIMEW                     0311
      PUNCH 40,NR,MODE,NMC,NLP,NEG,LZ,NZONE,NQS,MCL,TIMEL                   0312
     1,BLANK,AST,TEE,PLUS,PERIOD,DASH,EQUAL,MINUS,FFF,UUU,PPP               0313
      PUNCH 41,NR,NMC,TIME,DT,FLEX,CS,CR,BR,RS                              0314
      PUNCH 42, NMC,NR,(NMC,R(K),U(K),V(K),Q(K),T(K),P(K),NR,K,K=1,100)     0315
      PUNCH 43,NMC,NR,(NMC,RZ(K),R2(K),ZMAS(K),DR(K),RHOZ(K)                0316
     1,TR(K),NR,K,K=1,100)                                                  0317
      PUNCH 50,NR,TIME,TEE,WKT,TDN2,TDO2                                    0318
      PUNCH 50,NR,SCALE,EQUAL,Q71,Q47,TIMEW                                 0319
      END FILE 16                                                           0320
1502  CALL UNLOAD(16)                                                       0321
      CEASE=1.0E+29                                                         0322
      END FILE 32                                                           0323
      END FILE 42                                                           0324
      REWIND 42                                                             0325
      CALL COPY(42,32)                                                      0326
      CALL UNLOAD(42)                                                       0327
                                                                            0328
C   EXTRA COPY OF USER TAPE 32 INFO COPIED AS FILE 2 OF TAPE 15             0329
      END FILE 32                                                           0330
      REWIND 32                                                             0331
      CALL COPY(32,15)                                                      0332
      END FILE 15                                                           0333
      CALL COPY (32,15)                                                     0334
      END FILE 15                                                           0335
      CALL UNLOAD(15)                                                       0336
      CALL UNLOAD(32)                                                       0337
      END FILE 41                                                           0338
      CALL UNLOAD(41)                                                       0339
      END FILE 31                                                           0340
      CALL UNLOAD(31)                                                       0341
1500  CALL EXIT                                                             0342
   65 NTI=0                                                                 0343
      IF(NMC-MCL)  64,64,52                                                 0344
64    IF(TIME-TIMEL)  66,66,52                                              0345
C SET RETARDED VARIABLES                                                    0346
66    DO 71  K=1,100                                                        0347
67    VR(K)=V(K)                                                            0348
68    ER(K)=E(K)                                                            0349
69    PR(K)=P(K)                                                            0350
70    W(K,4)=T(K)                                                           0351
      W(K,6)=V(K)                                                           0352
      W(K,7)=P(K)                                                           0353
      W(K,8)=Q(K)                                                           0354
      W(K,9)=E(K)                                                           0355
71    DIVFR(K)=DIVFA(K)                                                     0356
      KRT=0                                                                 0357
```

```
72    IF(DT-50.0*DTMIN)  74,74,73                                    0358
73    DTMIN=DT/50.0                                                  0359
74    CALL HYDRO                                                     0360
      KZ4=KZ4                                                        0361
      IF(U(99)-1.0E+03) 290,290,52                                   0362
C  INITIAL TEMPERATURE EXTRAPOLATION                                 0363
290   CONTINUE                                                       0364
C  ITERATION CYCLE RE-ENTRY POINT                                    0365
C DERIVATIVES  WITH RESPECT TO TEMP     DP      DE                   0366
3000  DO 301    K=1,LZ                                               0367
3001  IF(T(K))  3002,3002,300                                        0368
3002  KZ2=-3000                                                      0369
      CALL DIAGNS                                                    0370
      GO TO 52                                                       0371
300   TP(K)=1.02*T(K)                                                0372
301   TS(K)=TP(K)                                                    0373
303   CALL STATE                                                     0374
304   DO 310  K=1,LZ                                                 0375
305   PP(K)=P(K)                                                     0376
306   EP(K)=E(K)                                                     0377
309   TM(K)=0.98*T(K)                                                0378
310   TS(K)=TM(K)                                                    0379
311   CALL STATE                                                     0380
312   DO  318  K=1,LZ                                                0381
313   PM(K)=P(K)                                                     0382
314   EM(K)=E(K)                                                     0383
315   DPT(K)=(PP(K)-PM(K))/(TP(K)-TM(K))                             0384
316   DET(K)=(EP(K)-EM(K))/(TP(K)-TM(K))                             0385
318   TS(K)=T(K)                                                     0386
320   CALL STATE                                                     0387
C  CALL RADIATIVE PROPERTIES ROUTINE                                 0388
      CALL SWABZ                                                     0389
501   LZR=LZR                                                        0390
      MCP=MCP                                                        0391
      KZ1=KZ1                                                        0392
      KZ2=KZ2                                                        0393
      KZ3=KZ3                                                        0394
      KZ8=KZ8                                                        0395
      KZ9=KZ9                                                        0396
321   NCW=NCW                                                        0397
322   DTAU(LZP1,1)=0.0                                               0398
                                                                     0399
C  INTENSITY INTEGRATION                                             0400
500   CALL FLUXS                                                     0401
502   IF(IPS)  503,503,530                                           0402
503   IPS=IPS+1                                                      0403
504   TIME=TIME-DT                                                   0404
505   GO TO 2000                                                     0405
C  TEMPERATURE TEST BYPASS ON FIRST GUESS                            0406
530   IF(NTI)  540,540,532                                           0407
C TEMPERATURE TEST                                                   0408
```

```
532   DO 536  K=1,LZ                                                    0409
533   IF(ABSF(DTM(K)/T(K))-0.1 )  536,536,540                          0410
536   CONTINUE                                                         0411
C  MAIN CYCLE COMPLETED --  RETURN TO 2000                            0412
537   GO TO 2000                                                       0413
540   NTI=NTI+1                                                        0414
C  ENERGY EQUATION  BLOCK 600                                         0415
600   IF(NTI-3) 601,601,2000                                           0416
601   DO 617 K=1,LZ                                                    0417
602   L=K                                                              0418
603   N=110+K                                                          0419
613   DE(K)=(E(K)-ER(K))/DT                                            0420
614   PA(K)=(PR(K)+P(K))/2.0                                           0421
615   RESDUE(K)=DE(K)+(PA(K)+Q(K))*(V(K)-VR(K))/DT                     0422
625  1+(DIVFA(K)+DIVFR(K))/2.0                                         0423
617   CONTINUE                                                         0424
C  TEMPERATURE ITERATION OF ENERGY EQUATION                           0425
700   IF(LZR-4)  703,703,701                                           0426
701   CALL COEFF                                                       0427
702   GO TO 829                                                        0428
C  NO ITERATION OF RADIATION                                          0429
703   DO 704 K=1,LZ                                                    0430
704   DTM(K)=-RESDUE(K)/((DET(K)/DT)+(DPT(K)/(2.0*DT))*(V(K)-VR(K)))   0431
705   IF(NMC-NLP)       906,706,906                                    0432
706   NLP=NLP+50                                                       0433
      PRINT 707,NMC,TIME,DT,LZ                                         0434
707   FORMAT(23H PROGRESS REPORT   NMC=I4, 5X,5HTIME=1PE12.3,5X,11HTIME 0435
707  1 STEP=1PE12.3,10X,5HNORAD I4///)                                 0436
708   GO TO 906                                                        0437
                                                                       0438
C   RADIATIVE ITERATION                                                0439
829   CONTINUE                                                         0440
830   IF(NMC-NLP)  987,831,987                                         0441
831   NLP=NLP+50                                                       0442
      PRINT 832,NMC,TIME,DT,LZ                                         0443
832   FORMAT(23H PROGRESS REPORT   NMC=I4, 5X,5HTIME=1PE12.3,5X,11HTIME 0444
832  1 STEP=1PE12.3,10X,8HRADHYDRO I4///)                              0445
C                                                                      0446
C     COEFFICIENTS NOW KNOWN-SOLUTION FOR DTM(K)                       0447
987   LZJ=LZR+1                                                        0448
      DO988  K=LZJ,LZ                                                  0449
988   DTM(K)=-RESDUE(K)/((DET(K)/DT)+(DPT(K)/(2.0*DT))*(V(K)-VR(K)))   0450
900   DTM(LZR)=(EN(LZR)-EN(LZR-1))/(CN(LZR)-CN(LZR-1))                 0451
902   DTM(LZR-1)=(EN(LZR-1)-CN(LZR-1)*DTM(LZR))                        0452
      DO    905    J=3,LZR                                             0453
      K=LZJ-J                                                          0454
905   DTM(K)=EN(K)-DN(K)*DTM(K+2)-CN(K)*DTM(K+1)                       0455
906   CONTINUE                                                         0456
C  STABILITY CHECK                                                    0457
907   DO 930  K=1,LZ                                                   0458
908   IF(DTM(K))  909,930,910                                          0459
```

```
909    IF(ABSF(DTM(K))-0.5*T(K))  930,930,918                                  0460
910    IF(T(K)-2.0E+05)  912,911,911                                           0461
911    IF(ABSF(DTM(K))-0.9*T(K))  930,930,918                                  0462
912    IF(ABSF(DTM(K))-5.0*T(K))  930,930,918                                  0463
918    KZ2=-920                                                                0464
919    KZ5=K                                                                   0465
920    CALL DIAGNS                                                             0466
921    KRT=KRT+1                                                               0467
922    IF(KRT-3)  923,933,933                                                  0468
923    DO 927  L=1,LZ                                                          0469
       U(L)=W(L,3)                                                             0470
       T(L)=W(L,4)                                                             0471
       R(L)=W(L,5)                                                             0472
       V(L)=W(L,6)                                                             0473
       P(L)=W(L,7)                                                             0474
       Q(L)=W(L,8)                                                             0475
       E(L)=W(L,9)                                                             0476
       R2(L)=R(L)**2                                                           0477
927    CONTINUE                                                                0478
928    WRITE OUTPUT TAPE 6,929,NMC,KRT,K                                       0479
       PRINT 929,NMC,KRT,K                                                     0480
929    FORMAT(4I8)                                                             0481
       NTI=0                                                                   0482
       GO TO 74                                                                0483
930    CONTINUE                                                                0484
C  MODIFY TEMPERATURE                                                          0485
933    DO 934  K=1,LZ                                                          0486
934    T(K)=T(K)+DTM(K)                                                        0487
935    GO TO 3000                                                              0488
936    END                                                                     0489
*      FORTRAN                                                                 0490
CF1SWZ                                                                         0491
       SUBROUTINE SWABZ                                                        0492
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM             0493
       D(1)=0.0                                                                0494
                                                                               0495
       DO 126  K=1,LZ                                                          0496
       DO 135  N=1,NSZ                                                         0497
       TS(K)=TS(K)/11606.5                                                     0498
       ETA(K)=1.293E-03*V(K)                                                   0499
       ALPHA=9.0+EXPF(-TIME/1.0E-03*(ETA(K)**2))                               0500
       BETA=((6.0E-18*V(K))**0.5)*(TS(K)**0.1)                                 0501
300    ZI(K,1)=EXPF(-1.5*(R(K)-R(K-1))    /((ETA(K)**1.5)/((TS(K)**ALPHA)      0502
300   2+BETA)         +0.2*(ETA(K)**1.91)/(TS(K)**2.73)                        0503
300   3+0.023*(ETA(K)**1.8)*(TS(K)**0.25)+1.0E-07*(ETA(K)**2.0)/               0504
300   4((TS(K)**(-5.0))+4.0E-14*V(K))))                                        0505
       ZI(K,1)=ZI(K,1)*0.99999998                                              0506
       ZI(K,2)=ZI(K,1)                                                         0507
       HMFP=(ETA(K)**1.5)/((TS(K)**ALPHA)+BETA)                                0508
      1+0.2*(ETA(K)**1.91)/(TS(K)**2.73)                                       0509
      2+0.023*(ETA(K)**1.8)*(TS(K)**0.25)                                      0510
```

```
     3+1.0E-07*(ETA(K)**2.0)*(TS(K)**5.00)                                    0511
      HMFF=9.3E-06*(ETA(K)**2.0)*(TS(K)**3.5)                                 0512
      IF(TS(K)-10.0)  301,298,298                                             0513
298   IF(HMFF-HMFP)  299,301,301                                              0514
299   HMFP=HMFF                                                               0515
301   DTAU(K,1)=1.5*(R(K)-R(K-1))/HMFP                                        0516
302   A(K,N)=EXPF(-DTAU(K,N))                                                 0517
312   BC(K,N)=5.67E-05*(T(K)**4)                                              0518
      TS(K)=TS(K)*11606.5                                                     0519
135   CONTINUE                                                                0520
126   CONTINUE                                                                0521
                                                                              0522
C     SOURCE FUNCTION INTERPOLATION                                           0523
      DO 201  K=1,LZ                                                          0524
      DO 202  N=1,NSZ                                                         0525
311   BB(K,N)=(((((R(K+1)-R(K))*T (K)**4)+((R(K)-R(K-1))*T (K+1)**4))/        0526
311  1(R(K+1)-R(K-1))))*5.67E-05                                              0527
      BB(K,2)=BB(K,1)                                                         0528
506   S(K,N)=(BC(K,N)-BC(K+1,N))/(DTAU(K,N)+DTAU(K+1,N))                      0529
507   W(K,N)=1.0-A(K,N)-A(K,N)*DTAU(K,N)                                      0530
508   IF(W(K,N)-1.0E-04)  509,202,202                                         0531
509   W(K,N)=0.5*(DTAU(K,N)**2)                                               0532
202   CONTINUE                                                                0533
201   CONTINUE                                                                0534
                                                                              0535
C     SET OPTICAL INDICES                                                     0536
      KM=0                                                                    0537
501   KD=0                                                                    0538
502   KN=0                                                                    0539
503   TAU(LZ,1)=DTAU(LZ,1)                                                    0540
511   DO  522  J=1,LZM1                                                       0541
512   K=LZ-J                                                                  0542
513   TAU(K,1)=TAU(K+1,1)+DTAU(K,1)                                           0543
514   IF(TAU(K,1)-5.0E-06)  522,515,515                                       0544
515   KN=KN+1                                                                 0545
516   IF(KN-1)  518,517,518                                                   0546
517   LZR=K+3                                                                 0547
518   IF(TAU(K,1)-.44) 522,519,519                                            0548
519   KD=KD+1                                                                 0549
520   IF(KD-1)  622,523,622                                                   0550
523   FBR=R(K)                                                                0551
622   IF(TAU(K,1)-1.0)  522,623,623                                           0552
623   KM=KM+1                                                                 0553
624   IF(KM-1)  522,625,522                                                   0554
625   MCP=K                                                                   0555
522   CONTINUE                                                                0556
525   IF(LZR-LZ+1) 530,530,529                                                0557
529   LZR=LZ-1                                                                0558
530   CONTINUE                                                                0559
      IF(KD)  521,521,524                                                     0560
521   FBR=0.0                                                                 0561
```

```
      MCP=0                                                              0562
524   CONTINUE                                                           0563
C  SET CUTOFF WAVELENGTH  CWL(K)                                        0564
      CWMAX=0.0                                                          0565
      KCWMAX=0                                                           0566
      CWL(MCP,1)=0.0                                                     0567
      NCW=MCP                                                            0568
      TR(MCP)=T(MCP)                                                     0569
                                                                         0570
540   DO  565  K=1,LZ                                                    0571
541   IF(T(K)-TDN2)  546,542,542                                         0572
                                                                         0573
C   OPACITY DUE TO ATOMIC SPECIES                                        0574
542   CWL(K,1)= 700.0*EXPF(-0.36*(T(K)-11606.5)/11606.5)                 0575
B544  DTH(K)=212121212121                                                0576
      IF(CWL(K,1)-275.0)   545,560,560                                   0577
545   CWL(K,1)=275.0                                                     0578
      GO TO 560                                                          0579
546   IF(T(K)-TDO2)  551,547,547                                         0580
                                                                         0581
C  OPACITY DUE TO N2 MOLECULE                                            0582
B 547 DTH(K)=454545454545                                                0583
      TCN2=5.0E-07*1.293E-03*V(K)                                        0584
549   CWL(K,1)=1140.0*(((R(K)-R(K-1))/(V(K)*1.293E-03))**0.11)           0585
     1*(1.0-EXPF(-TIME/TCN2))                                            0586
      IF(CWL(K,1)-1000.0)  550,560,560                                   0587
550   CWL(K,1)=1000.0                                                    0588
      GO TO 560                                                          0589
                                                                         0590
C   OPACITY DUE TO O2 MOLECULE                                           0591
551   TC=4250.0-271.0*LOGF(1.293E-03* V(K))                              0592
      TCO2=3.0E-07*1.293E-03*V(K)                                        0593
B 552 DTH(K)=676767676767                                                0594
553   IF(T(K)-TC)554,554,556                                             0595
554   CWL(K,1)=1500.0+TS(K)*(0.163+0.0745*LOGF(DR(K)*RHOZ(K)/1.293E-03))  0596
554  1*(1.0-EXPF(-TIME/TCO2))                                            0597
555   GO TO 557                                                          0598
556   TSB=TS(K)*(.0647*LOGF(R(K)-R(K-1))-0.25-.109*LOGF(1.293E-03*V(K)))  0599
      CWL(K,1)=3500.0+TSB-(2000.0+TSB)*EXPF(-TIME/TCO2)                  0600
557   IF(CWL(K,1)-1500.0) 558,560,560                                    0601
558   CWL(K,1)=1500.0                                                    0602
560   IF(K-MCP)  565,565,561                                             0603
561   IF(CWL(K,1)-CWMAX)  565,565,563                                    0604
563   CWMAX=CWL(K,1)                                                     0605
564   KCWMAX=K                                                           0606
565   CONTINUE                                                           0607
566   NCW=KCWMAX                                                         0608
                                                                         0609
      IF(LZR-NCW)   603,603,604                                          0610
603   LZR=NCW+1                                                          0611
C   CALCULATE ZO VALUES                                                  0612
```

```
604    ZO(1,1)=ZI(1,1)                                                        0613
       KD=0                                                                   0614
605    DO 670  L=2,LZ                                                         0615
       TR(L)=T(L)                                                             0616
       IF(T(L)-8.0E+04)  607,668,668                                          0617
607    KD=KD+1                                                                0618
       IF(KD-1)  608,608,609                                                  0619
608    MS=L-1                                                                 0620
609    IF(DTAU(L,1)-1.0)  610,668,668                                         0621
C    SELECT TR(L)= TEMPERATURE OF RADIATION TRAVERSING ZONE L                 0622
610    LIM=L-1                                                                0623
611    TAUSUM=0.0                                                             0624
615    DO 630 NN=1,LIM                                                        0625
616    M=L-NN                                                                 0626
617    TAUSUM=TAUSUM+DTAU(M,1)                                                0627
620    IF(TAUSUM-0.7)  630,627,627                                            0628
627    TR(L)=T(M)                                                             0629
628    IF(M-2)  630,630,638                                                   0630
630    CONTINUE                                                               0631
       IF(T(L)-2500.0)  631,632,632                                           0632
631    TR(L)=TR(L-1)                                                          0633
       GO TO 637                                                              0634
632    EPTSUM=0.0                                                             0635
       EPSUM=0.0                                                              0636
       DO 635  K=3,LIM                                                        0637
       EPSUM=EPSUM+DTAU(K,1)                                                  0638
       EPTSUM=EPTSUM+T(K)*DTAU(K,1)                                           0639
       IF(T(K)-2500.0)   636,636,635                                          0640
635    CONTINUE                                                               0641
636    TR(L)=EPTSUM/EPSUM                                                     0642
637    M=2                                                                    0643
638    IZ=M+1                                                                 0644
639    DO 646    I=IZ,L                                                       0645
640    IF(TR(L)*CWL(I,1)-2.0E+08)  641,641,644                                0646
641    ZI(I,2)= EXPF((-3.41E-08*CWL(I,1)*TR( L )+2.5E-25*((CWL(I,1)*TR(L      0647
641   2))**3))*EXPF(-3.07E+13/((CWL(I,1)*TR( L ))**1.8)))                     0648
642    GO TO 646                                                              0649
644    ZI(I,2)=7.61E+22/((CWL(I,1)*TR( L ))**3)                               0650
646    CONTINUE                                                               0651
       ZMAX=ZI(M+1,2)                                                         0652
       ZO(L,1)=ZI(M+1,2)                                                      0653
647    IF(L-M-1) 668,663,650                                                  0654
C    CORRECTION FOR INTERMEDIATE ZONES                                        0655
650    IZ=M+2                                                                 0656
651    CWLTST=CWL(M+1,1)                                                      0657
652    DO 660  NZ=IZ,L                                                        0658
653    IF(CWL(NZ,1)-CWLTST)  654,654,657                                      0659
654    IF(NZ-L)  660,655,668                                                  0660
655    ZO(L,1)=1.0                                                            0661
656    GO TO 660                                                              0662
657    ZO(L,1)=ZI(NZ,2)/ZMAX                                                  0663
```

```
658    CWLTST=CWL(NZ,1)                                                    0664
659    ZMAX=ZI(NZ,2)                                                       0665
660    CONTINUE                                                            0666
663    IF(ZO(L,1)-ZI(L,1))  670,668,668                                    0667
668    ZO(L,1)=ZI(L,1)                                                     0668
670    CONTINUE                                                            0669
671    DO 676  K=1,LZ                                                      0670
672    IF(DTAU(K,1)-1.0E-04)  673,675,675                                  0671
673    A(K,1)=DTAU(K,1)                                                    0672
674    GO TO 676                                                           0673
675    A(K,1)=1.0-A(K,1)                                                   0674
676    CONTINUE                                                            0675
       DO 699  K=1,LZ                                                      0676
       IF(S(K,1)-1.69E+38)  767,767,677                                    0677
767    HZ(K)=BB(K,1)*A(K+1,1)                                              0678
       DZ(K)=2.0*S(K,1)*W(K+1,1)                                           0679
       HP(K)=BC(K+1,1)*A(K+1,1)                                            0680
       IF(HZ(K)-DZ(K)-HP(K)*0.01)    677,677,699                           0681
677    S(K,1)=0.0                                                          0682
678    BB(K,1)=BC(K+1,1)                                                   0683
679    BB(K,2)=BC(K,1)                                                     0684
699    CONTINUE                                                            0685
900    IF(BC(1,1)-BC(2,1))   915,916,916                                   0686
915    BB(1,2)=BC(1,1)                                                     0687
916    CONTINUE                                                            0688
       EMS=ZI(NCW,2)                                                       0689
179    RETURN                                                              0690
       END                                                                 0691
                                                                           0692
*      FORTRAN                                                             0693
CF1RJST                                                                    0694
       SUBROUTINE REJUST                                                   0695
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM         0696
       TDO2=7100.0*EXPF((0.43429*LOGF(RHOZ(99)/1.293E-03))/5.3)            0697
       TDN2=15000.0*EXPF((0.43429*LOGF(RHOZ(99)/1.293E-03))/4.8)           0698
       RETURN                                                              0699
       END                                                                 0700
*      FORTRAN                                                             0701
CF1RZNE                                                                    0702
       SUBROUTINE REZONE                                                   0703
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM         0704
                                                                           0705
1      M=LZP2                                                              0706
       ZETA=(T(M-1)+T(M))/(E(M-1)+E(M))                                    0707
C  ROUTINE TO COMBINE ZONES M AND M-1                                      0708
       FKE= (ZMAS(M-2)*(U(M-2)**2)+ZMAS(M-1)*((U(M-1)**2)+(U(M-2)**2)      0709
      2)+ZMAS(M)*((U(M-1)**2)+(U(M)**2))+ZMAS(M+1)*(U(M)**2))              0710
       FMV=(ZMAS(M-2)*U(M-2)+ZMAS(M-1)*(U(M-1)+U(M-2))+ZMAS(M)*(U(M-1)      0711
      2+U(M))+ZMAS(M+1)*U(M))                                              0712
       FKA=ZMAS(M-2)+ZMAS(M-1)+ZMAS(M)                                     0713
       FKB=ZMAS(M-1)+ZMAS(M)+ZMAS(M+1)                                     0714
```

```
      FKB=FKB/FKA                                                          0715
      FKE=FKE/FKA                                                          0716
      FMV=FMV/FKA                                                          0717
      FSR=(FKE*FKB)*(FKB+1.0)-FKB*(FMV**2)                                 0718
      IF(FSR)     7700,7701,7701                                           0719
7700  U(M-2)=U(M-2)                                                        0720
      U(M-1)=U(M)                                                          0721
      GO TO 7733                                                           0722
7701  FSR=(FSR**0.5)/FKB                                                   0723
      UPLUS=(FMV+FSR)/(FKB+1.0)                                            0724
      UMINS=(FMV-FSR)/(FKB+1.0)                                            0725
      IF(U(M-2)-U(M))  7722,7711,7711                                      0726
7711  U(M-2)=UPLUS                                                         0727
      U(M-1)=UMINS                                                         0728
      GO TO 7733                                                           0729
 7722 U(M-2)=UMINS                                                         0730
      U(M-1)=UPLUS                                                         0731
7733  CONTINUE                                                             0732
2     E(M-1)=(ZMAS(M-1)*E(M-1)+ZMAS(M)*E(M))/(ZMAS(M-1)+ZMAS(M))           0733
3     T(M-1)=(E(M-1)      *ZETA)                                           0734
4     ZMAS(M-1)=ZMAS(M-1)+ZMAS(M)                                          0735
5     DIVFA(M-1)=(R2(M)/ZMAS(M-1))*(FOS(M)-FIS(M))                         0736
5    1-(R2(M-2)/ZMAS(M-1))*(FOS(M-2)-FIS(M-2))                             0737
7     R(M-1)=R(M)                                                          0738
8     RZ(M-1)=RZ(M)                                                        0739
9     R2(M-1)=R2(M)                                                        0740
10    DR(M-1)=DR(M-1)+DR(M)                                                0741
11    RR(M-1)=RR(M)                                                        0742
13    TR(M-1)=TR(M)                                                        0743
14    V(M-1)=(R(M-1)**3-R(M-2)**3)/(3.0*ZMAS(M-1))                         0744
15    P(M-1)=(P(M-1)+P(M))/2.0                                             0745
16    Q(M-1)=(Q(M-1)+Q(M))/2.0                                             0746
      DTM(M-1)=0.0                                                         0747
                                                                           0748
C   SHIFT IN EXTERIOR ZONES                                                0749
20    DO 50  K=M,99                                                        0750
21    V(K)=V(K+1)                                                          0751
22    T(K)=T(K+1)                                                          0752
23    RHOZ(K)=RHOZ(K+1)                                                    0753
24    U(K)=U(K+1)                                                          0754
25    P(K)=P(K+1)                                                          0755
26    R(K)=R(K+1)                                                          0756
27    R2(K)=R2(K+1)                                                        0757
28    RZ(K)=RZ(K+1)                                                        0758
29    DR(K)=DR(K+1)                                                        0759
30    Q(K)=Q(K+1)                                                          0760
31    E(K)=E(K+1)                                                          0761
32    DIVFA(K)=DIVFA(K+1)                                                  0762
33    UR(K)=UR(K+1)                                                        0763
34    ZMAS(K)=ZMAS(K+1)                                                    0764
35    RR(K)=RR(K+1)                                                        0765
```

```
      DTM(K)=DTM(K+1)                                                    0766
50    TR(K)=TR(K+1)                                                      0767
                                                                         0768
C  ADDITION OF NEW ZONE 100                                             0769
51    R(100)=R(99)+DR(99)                                                0770
52    R2(100)=R(100)**2                                                  0771
53    RZ(100)=R(100)                                                     0772
54    RHOZ(100)=RHOZ(99)                                                 0773
55    RR(100)=1.0                                                        0774
56    DR(100)=DR(99)                                                     0775
57    ZMAS(100)=(RHOZ(100)/3.0)*((RZ(100)**3)-(RZ(99)**3))               0776
58    U(100)=0.0                                                         0777
59    T(100)=T(99)                                                       0778
60    V(100)=V(99)                                                       0779
62    Q(100)=0.0                                                         0780
                                                                         0781
      T(101)=T(100)                                                      0782
      RETURN                                                             0783
      END                                                                0784
*     FORTRAN                                                            0785
      SUBROUTINE SPLIT                                                   0786
C    STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM        0787
      M=100-KZ2                                                          0788
      DO 30  J=1,M                                                       0789
      K=101-J                                                            0790
      R(K)=R(K-1)                                                        0791
      DR(K)=DR(K-1)                                                      0792
      RR(K)=RR(K-1)                                                      0793
      RHOZ(K)=RHOZ(K-1)                                                  0794
      RZ(K)=RZ(K-1)                                                      0795
      R2(K)=R2(K-1)                                                      0796
      U(K)=U(K-1)                                                        0797
      RR(K)=RR(K-1)                                                      0798
      TR(K)=TR(K-1)                                                      0799
      V(K)=V(K-1)                                                        0800
      P(K)=P(K-1)                                                        0801
      ZMAS(K)=ZMAS(K-1)                                                  0802
      Q(K)=Q(K-1)                                                        0803
      T(K)=T(K-1)                                                        0804
      E(K)=E(K-1)                                                        0805
      DIVFA(K)=DIVFA(K-1)                                                0806
      UR(K)=UR(K-1)                                                      0807
30    CONTINUE                                                           0808
C   ADD ZONE JUST OUTSIDE K=KZ2                                          0809
      ZMAS(KZ2)=ZMAS(KZ2)/2.0                                            0810
          ZMAS(KZ2+1)=ZMAS(KZ2)                                          0811
      V(KZ2+1)=V(KZ2)                                                    0812
      RHOZ(KZ2+1)=RHOZ(KZ2)                                              0813
      R(KZ2)=((R(KZ2+1)**3)-(3.0*ZMAS(KZ2+1)*   V(KZ2+1)))**(1.0/3.0)    0814
      RZ(KZ2)=((RZ(KZ2+1)**3)-(3.0*ZMAS(KZ2+1)/RHOZ(KZ2+1)))**(1.0/3.0)  0815
      RR(KZ2)=R(KZ2)/RZ(KZ2)                                             0816
```

```
      DR(KZ2+1)=R(KZ2+1)-R(KZ2)                                          0817
      DR(KZ2)=R(KZ2)-R(KZ2-1)                                            0818
      R2(KZ2)=R(KZ2)**2                                                  0819
      U(KZ2)=(((U(KZ2-1)**2)+(U(KZ2+1)**2))/2.0)**0.5                    0820
      TR(KZ2+1)=TR(KZ2)                                                  0821
      P(KZ2+1)=P(KZ2)                                                    0822
      Q(KZ2+1)=Q(KZ2)                                                    0823
      T(KZ2+1)=0.9*T(KZ2)                                                0824
      T(KZ2)=1.1*T(KZ2)                                                  0825
      E(KZ2+1)=0.85*E(KZ2)                                               0826
      E(KZ2)=1.15*E(KZ2)                                                 0827
      DIVFA(KZ2+1)=DIVFA(KZ2)                                            0828
      UR(KZ2)=0.5*(UR(KZ2-1)+UR(KZ2+1))                                  0829
      RETURN                                                             0830
      END                                                                0831
*     FORTRAN                                                            0832
CF1CGSPO     CGS PRINT OUT PACKAGE   FIRE 1                              0833
      SUBROUTINE CGSPO                                                   0834
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM       0835
45    FORMAT(1H1//39H AT COMPLETION OF MASTER CYCLE NUMBER  I4.25X.I5.   0836
45   240X.3HSE=0PF10.3//6H TIME=1PE10.3.21X.6HPOWER=1PE10.3.2X.5HWATTS.  0837
45   313X.10HTIME STEP=1PE10.3.20X.14HPASSES ENG EQ I4//                 0838
45   43H  K.2X.6HRADIUS.5X.8HPART VEL.2X.4HPRES.6X.3HQ/P.3X.3HTEV.4X.3HR 0839
45   5H0.7X.3HE/G.7X.4HFOUT.5X.5HFI/FO.2X.4HDIVF.5X.4HTEMP.4X.5HGAMMA.   0840
45   6X.4HPART.X.1HC.2X.3HTAU.5X.4HDTAU//(I4.1PE10.3.1P2E10.2.0P2F6.2.   0841
45   71P3E10.2.0PF6.2.1P2E10.2.0PF5.2.0PF6.2.1A1.0PF7.2.1A1.1PE9.2))     0842
46    FORMAT(1H /8H T PART=1PE12.4.9H FRACT    .5X.6HDTHYD=1PE12.4.8H SEC 0843
46   1S    .5X.6HPOW03=1PE12.4.8H WATTS  .5X.5HTEFF=0PF6.0.8H DEG K  .5X. 0844
46   24HLZR=I4/8H TYIELD=1PE12.4.9H ERGS     .5X.6HDTRAD=1PE12.4.8H SECS  0845
46   3  .5X.6HPOW34=1PE12.4.8H WATTS  .5X.5HTCOL=0PF6.0.8H DEG K  .5X.    0846
46   44HNCW=I4/8H IN ENG=1PE12.4.9H ERGS     .5X.6HDTMIN=1PE12.4.8H SECS  0847
46   5  .5X.6HPOW45=1PE12.4.8H WATTS  .5X.5HWLMX=0PF6.0.8H ANG     .5X.    0848
46   64HOD1=I4/8H KN ENG=1PE12.4.9H ERGS     .5X.6HDTLST=1PE12.4.8H SECS  0849
46   7  .5X.6HPOW57=1PE12.4.8H WATTS  .5X.5HTY41=0PF7.2. 7HKT      .5X.    0850
46   8.4HEMS=0PF5.4)                                                     0851
36    FORMAT(8H TOT ES=1PE12.4.9H ERGS     5X.6HF8RAD=1PE12.4.8H CM      . 0852
36   25X.6HPOW71=1PE12.4.8H WATTS  .5X.5HTY71=0PF7.2. 7HKT      .5X.      0853
36   34HNSZ=I4/8H E AMBT=1PE12.4.9H ERGS     5X.6HSHRAD=1PE12.4.8H CM     0854
36   4 .5X.6HPOW47=1PE12.4.8H WATTS  .5X.5HTY47=0PF7.2. 7HKT      .5X.     0855
36   56HD1589-I2)                                                        0856
47    FORMAT(1H1//.                                                      0857
47   43H  K.2X.6HRADIUS.5X.8HPART VEL.2X.4HPRES.6X.3HQ/P.3X.3HTEV.4X.3HR 0858
47   5H0.7X.3HE/G.7X.4HFOUT.5X.5HFI/FO.2X.4HDIVF.5X.4HTEMP.4X.5HGAMMA.   0859
47   6X.4HPART.X.1HC.2X.3HTAU.5X.4HDTAU//(I4.1PE10.3.1P2E10.2.0P2F6.2.   0860
47   71P3E10.2.0PF6.2.1P2E10.2.0PF5.2.0PF6.2.1A1.0PF7.2.1A1.1PE9.2))     0861
      YIELD=EQUAL                                                        0862
      DO 1000 K=4.LZ                                                     0863
      IF(T(K-1)-0.6*T(K))  1002.1002.1000                                0864
1002  KZ2=0                                                              0865
      CALL DIAGNS                                                        0866
1000  CONTINUE                                                           0867
```

```
      RSHOCK=0.0                                                        0868
      SAVE1=DT                                                          0869
      DT=CN(LZ)                                                         0870
      STX=P(LZ)                                                         0871
      POWER=TFLUX*1.0E-07                                               0872
      TCOL=(TFLUX/(  EMS    *12.56*5.67E-05*(FBR**2)))**0.25            0873
      TEFF=(TFLUX/(12.56*5.67E-05*(FBR**2)))**0.25                      0874
      CALL PHOTOG                                                       0875
C   ESTIMATE OF OBSERVED SPECTRAL DISTRIBUTION                          0876
      IF(FBR)  1059,1059,1061                                           0877
1059  P47=0.37*POWER                                                    0878
      Q47=Q47+P47*(TIME-TIMES)/4.186E+12                                0879
      Q41=0.0                                                           0880
      P03=0.0                                                           0881
      P34=0.0                                                           0882
      P45=0.0                                                           0883
      P57=0.0                                                           0884
      P71=0.0                                                           0885
      Q71=0.0                                                           0886
1060  GO TO 1600                                                        0887
1061  SAVET=T(MCP)                                                      0888
1062  T(MCP)=TCOL                                                       0889
1064  IF( T(MCP)*CWLM  -2.0E+08)  1065,1065,1068                        0890
1063  CWLM=CWL(NCW,1)                                                   0891
1065  FLM = EXPF((-3.41E-08*CWLM  * T(MCP)+2.5E-25*((CWLM  * T(MCP))**3)  0892
1065 1)*EXPF(-3.07E+13/((CWLM  * T(MCP))**1.8)))                        0893
1067  GO TO 1164                                                        0894
1068  FLM =7.61E+22/((CWLM  *T(MCP))**3)                                0895
1164  IF( T(MCP)*3000.0-2.0E+08)  1165,1165,1168                        0896
1165  F3  = EXPF((-3.41E-08*3000.0* T(MCP)+2.5E-25*((3000.0* T(MCP))**3)  0897
1165 1)*EXPF(-3.07E+13/((3000.0* T(MCP))**1.8)))                        0898
1167  GO TO 1264                                                        0899
1168  F3  =7.61E+22/((3000.0*T(MCP))**3)                                0900
1264  IF( T(MCP)*4000.0-2.0E+08)  1265,1265,1268                        0901
1265  F4  = EXPF((-3.41E-08*4000.0* T(MCP)+2.5E-25*((4000.0* T(MCP))**3)  0902
1265 1)*EXPF(-3.07E+13/((4000.0* T(MCP))**1.8)))                        0903
1267  GO TO 1364                                                        0904
1268  F4  =7.61E+22/((4000.0*T(MCP))**3)                                0905
1364  IF( T(MCP)*5000.0-2.0E+08)  1365,1365,1368                        0906
1365  F5  = EXPF((-3.41E-08*5000.0* T(MCP)+2.5E-25*((5000.0* T(MCP))**3)  0907
1365 1)*EXPF(-3.07E+13/((5000.0* T(MCP))**1.8)))                        0908
1367  GO TO 1464                                                        0909
1368  F5  =7.61E+22/((5000.0*T(MCP))**3)                                0910
1464  IF( T(MCP)*7000.0-2.0E+08)  1465,1465,1468                        0911
1465  F7  = EXPF((-3.41E-08*7000.0* T(MCP)+2.5E-25*((7000.0* T(MCP))**3)  0912
1465 1)*EXPF(-3.07E+13/((7000.0* T(MCP))**1.8)))                        0913
1467  GO TO 1564                                                        0914
1468  F7  =7.61E+22/((7000.0*T(MCP))**3)                                0915
1564  IF( T(MCP)*10000.0-2.0E+08)  1565,1565,1568                       0916
1565  F1= EXPF((-3.41E-08*10000.0* T(MCP)+2.5E-25*((10000.0* T(MCP))**3)  0917
1565 1)*EXPF(-3.07E+13/((10000.0* T(MCP))**1.8)))                       0918
```

```
1567  GO TO 1580                                                              0919
1568  F1  =7.61E+22/((10000.0*T(MCP))**3)                                     0920
1580  CONTINUE                                                                0921
      T(MCP)=SAVET                                                            0922
1581  IF(CWLM-3000.0) 1583,1583,1582                                          0923
1582  F3=FLM                                                                  0924
1583  IF(CWLM-4000.0) 1585,1585,1584                                          0925
1584  F4=FLM                                                                  0926
1585  IF(CWLM-5000.0) 1587,1587,1586                                          0927
1586  F5=FLM                                                                  0928
1587  IF(CWLM-7000.0) 1589,1589,1588                                          0929
1588  F7=FLM                                                                  0930
1589  IF(CWLM-10000.0) 1591,1591,1590                                         0931
1590  F1=FLM                                                                  0932
1591  CONTINUE                                                                0933
      P03=POWER*(FLM-F3)/FLM                                                  0934
      P34=POWER*(F3-F4)/FLM                                                   0935
      P45=POWER*(F4-F5)/FLM                                                   0936
      P57=POWER*(F5-F7)/FLM                                                   0937
      P71=POWER*(F7-F1)/FLM                                                   0938
      P47=POWER*(F4-F7)/FLM                                                   0939
      Q71=Q71+P71*(TIME-TIMES)/4.186E+12                                      0940
      Q47=Q47+P47*(TIME-TIMES)/4.186E+12                                      0941
      Q41=Q47+Q71                                                             0942
                                                                              0943
1600  EN(1)=4.189*(R(1)**3)*(E(1)/V(1) )                                      0944
      BN(1)=EN(1)                                                             0945
      CN(1)=3.14159*(U(1)**2)*ZMAS(1)                                         0946
      DN(1)=CN(1)                                                             0947
      DPP(1)=BN(1)+DN(1)                                                      0948
      DO 490  K=2,LZ                                                          0949
      EN(K)=4.189*((R(K)**3)-(R(K-1)**3))*(E(K)/V(K))                         0950
      CN(K)=3.14159*(U(K)**2+U(K-1)**2)*ZMAS(K)                               0951
      BN(K)=BN(K-1)+EN(K)                                                     0952
      DN(K)=DN(K-1)+CN(K)                                                     0953
8     THETA(K)=212121212121                                                   0954
      IF(Q(K)-STX)  490,490,488                                               0955
488   STX=Q(K)                                                                0956
      J3=K-3                                                                  0957
489   RSHOCK=R(K)                                                             0958
490   DPP(K)=BN(K)+DN(K)                                                      0959
      EAMB=4.189*(R(LZ)**3)*E(99)/V(99)                                       0960
      ESYSM=DPP(LZ)-EAMB                                                      0961
      ETOTAL=ESYSM/4.186E+19                                                  0962
      PARTT=FLEX/YIELD                                                        0963
491   IF(NMC)  493,492,493                                                    0964
492   YIELD=ESYSM                                                             0965
      EQUAL=YIELD                                                             0966
493   CONTINUE                                                                0967
      WRITE OUTPUT TAPE22,495,NR,NMC,TIME,EAMB,NCW,MCP,DPP(NCW),DPP(MCP)      0968
     1,TR(NCW),TR(MCP),(K,EN(K),BN(K),CN(K),DN(K),DPP(K),A(K,1),ZO(K,1),      0969
```

```
     2ZI(K,1),TR(K),K=1,LZ)                                                    0970
495   FORMAT(1H1,13H ENERGY CHECK,10X,4HRUN=I4,5X,4HNMC=I4,5X,5HTIME=1PE       0971
495  210.3,5X,6HE AMB=1PE10.3//5H NCW=I4,5X,4HMCP=I4,5X,9HDPP(NCW)=1PE12      0972
495  3.4,5X, 9HDPP(MCP)=1PE12.4,5X,8HTR(NCW)=1PE10.3,5X,8HTR(MCP)=1PE10       0973
495  4.3//5H ZONE,3X,2HDE,10X,4HSUME,6X,3HDKE,7X,5HSUMKE,5X,9HSUMTOTALE,      0974
495  511X,1HA,11X,2HZO,10X,2HZI,10X,2HTR//(I5,1P5E12.4,3X,1P4E12.4))          0975
8494  THETA(1)=646464646464                                                   0976
8     THETA(2)=232323232323                                                   0977
      DO 29  K=1,LZ                                                           0978
      HZ(K)=DTAU(K,1)/1.5                                                     0979
      DMM(K)=DTAU(K,1)/(R(K)-R(K-1))                                          0980
      FZ(K)=4.28 E-08*(E(K)+P(K)*V(K))                                        0981
      DZ(K)=TAU(K,1)/1.5                                                      0982
      DP(K)=Q(K)/P(K)                                                         0983
      FM(K)=T(K)/11606.5                                                      0984
      HP(K)=FOS(K)*1.0E-07                                                    0985
      HPP(K)=ABSF(FIS(K)/FOS(K))                                              0986
      GM(K)=1.0+(P(K)*V(K)/E(K))                                              0987
      UR(K)=1.0/V(K)                                                          0988
      IF(PART(K)-4.0)  29,28,28                                               0989
828   DTH(K)=313131313131                                                     0990
29    CONTINUE                                                                0991
      SHVEL=U(J3)*(1.0+GM(J3))/2.0                                            0992
      PSHK=(2.0/(GM(J3)+1.0))*RHOZ(LZ)*(SHVEL**2)                             0993
      ESHK=0.5*((2.0*SHVEL/(1.0+GM(J3)))**2)                                  0994
      TRSHK=T(J3)*ESHK/E(J3)                                                  0995
      ETOE=(5.67E-05*(TRSHK**4)/(RHOZ(LZ)*SHVEL))+E(LZ)                       0996
      TTOE=ETOE*T(LZ)/E(LZ)                                                   0997
      POWSK=12.56*(R(J3+2)**2)*5.67E-05*(TRSHK**4)*FLM                        0998
      WRITE OUTPUT TAPE 6,496,NR,NMC,TIME,SHVEL,PSHK,TRSHK,TTOE               0999
      PUNCH 496, NR,NMC,TIME,SHVEL,PSHK,TRSHK,TTOE                            1000
496   FORMAT(2I4,1P5E12.4)                                                    1001
80    IF(LZ-40)  30,30,81                                                     1002
81    WRITE OUTPUT TAPE NTAPE,45,NMC,LZ,ETOTAL,TIME,POWER,DT,NTI,             1003
81   1(K,R(K),U(K),P(K),DP(K),FM(K),UR(K),E(K),HP(K),HPP(K),DIVFA(K),T(K      1004
81   2),GM(K),PART(K),THETA(K),DZ(K),DTH(K),HZ(K),K=1,40)                     1005
82    WRITE OUTPUT TAPE NTAPE,47,                                             1006
82   1(K,R(K),U(K),P(K),DP(K),FM(K),UR(K),E(K),HP(K),HPP(K),DIVFA(K),T(K      1007
82   2),GM(K),PART(K),THETA(K),DZ(K),DTH(K),HZ(K),K=41,LZ)                    1008
83    GO TO 85                                                                1009
30    WRITE OUTPUT TAPE NTAPE,45,NMC,LZ,ETOTAL,TIME,POWER,DT,NTI,             1010
30   1(K,R(K),U(K),P(K),DP(K),FM(K),UR(K),E(K),HP(K),HPP(K),DIVFA(K),T(K      1011
30   2),GM(K),PART(K),THETA(K),DZ(K),DTH(K),HZ(K),K=1,LZ)                     1012
85    WRITE OUTPUT TAPE NTAPE,46,PARTT,RJ,P03,TEFF,LZR,FLEX,RX,P34,TCOL,      1013
85   2NCW,BN(LZ),DTMIN,P45,CWL(NCW,1),MCP,DN(LZ),SAVE1,P57,Q41,EMS            1014
      WRITE OUTPUT TAPE NTAPE,36,DPP(LZ),FBR,P71,Q71,NSZ,EAMB,RSHOCK,P47      1015
     1,Q47,NR                                                                 1016
      W(1,9)=RSHOCK                                                           1017
      DT=SAVE1                                                                1018
      IF(NMC-1)  403,400,400                                                  1019
400   IF(NMC-10)   411,411,403                                                1020
```

```
403    CONTINUE                                                          1021
C  USER TAPE PRINT OUT                                                  1022
       WRITE OUTPUT TAPE 32,300,NR,NMC,TIME,LZ,P(99),T(99),RHOZ(99),POWER 1023
      2,P47,FBR,(K,R(K),U(K),P(K),UR(K),T(K),FOS(K),FIS(K),DMM(K),Q(K),  1024
      3FZ(K),E(K),HPPP(K),K=1,LZ)                                        1025
300    FORMAT(1H1,2I4,1PE12.3,I5,1P6E12.3/(I4,1P12E10.3))                1026
       WRITE OUTPUT TAPE 42,437,NR,NMC,TIME,T(1),TCOL,TEFF,ETOTAL,PARTT, 1027
      1BN(LZ),DN(LZ),ESYSM,EAMB,FLEX,Q41,Q47,Q71,P03,P34,P45,P57,P71,P47, 1028
      2FBR,RSHOCK                                                        1029
437    FORMAT(1H ,2I4,1P10E11.3/1P12E11.4///)                            1030
410    IF(NMC)  500,420,411                                              1031
411    IF(TIME-6.82E-05)  500,412,412                                    1032
412    NSIX=NSIX+1                                                       1033
413    IF(NSIX-3)  500,420,500                                           1034
420    NSIX=0                                                            1035
421    CALL SIXDP0                                                       1036
500    RETURN                                                            1037
       END                                                               1038
*      FORTRAN                                                           1039
CF1SIXD                                                                  1040
       SUBROUTINE SIXDP0                                                 1041
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM       1042
       FLAG=1.0E+31                                                      1043
       SOUNDS=SQRTF(1.4*P(99)/RHOZ(99))                                  1044
       DO 10  K=1,LZ                                                     1045
       BN(K)=P(K)/P(99)                                                  1046
       CN(K)=V(99)/V(K)                                                  1047
       DN(K)=T(K)/T(99)                                                  1048
       EN(K)=U(K)/SOUNDS                                                 1049
C     AVERAGE ZONE RADIUS IN FEET                                        1050
10     DP(K)=(R(K)+R(K-1))/60.96                                         1051
       WRITE TAPE 31,NR,TIME,LZ,(DP(K),BN(K),CN(K),DN(K),EN(K),GM(K),    1052
      1K=1,LZ),FLAG                                                      1053
       WRITE OUTPUT TAPE 41,50,NR,TIME,LZ,(DP(K),BN(K),CN(K),DN(K),EN(K), 1054
      1GM(K),K=1,LZ)                                                     1055
50     FORMAT(1H1/I4,X,27H SIXDPLOT DIAGNOSTICS TIME=1PE10.3,5X,3HLZ=I4  1056
50    2///(1P6E16.4))                                                    1057
       KZ6=KZ6+1                                                         1058
20     RETURN                                                            1059
       END                                                               1060
*      FORTRAN                                                           1061
CF1PHOTOG    PHOTOGRAPHIC BRIGHTNESS ROUTINE                             1062
       SUBROUTINE PHOTOG                                                 1063
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM       1064
       IF(NMC)  32,30,32                                                 1065
30     JTAPE=25                                                          1066
31     GO TO 40                                                          1067
32     IF(NMC- 3)  33,33,30                                              1068
33     JTAPE=6                                                           1069
40     CONTINUE                                                          1070
C    SOURCE FUNCTION EM(K) AND ABSORPTION COEFF BN(K)                    1071
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```
      DO 5  K=1,LZ                                                          1072
      BN(K)=(V(K)/(                                                         1073
     16.5*((1.293E-03*V(K))**1.15)/(((T(K)/11606.5)**8.0)+2.6E-12*V(K))     1074
     2+0.180*((1.293E-03*V(K))**1.91)/((T(K)/11606.5)**(3.5))               1075
     4+6.00E-05*((1.293E-03*V(K))**2.0)*((T(K)/11606.5)**0.59)              1076
     5+1.00E-09*((1.293E-03*V(K))**2.0)*((T(K)/11606.5)**4.00)))            1077
     6*(1.0-EXPF(-3.2E+04/T(K)))                                            1078
      TEA=3.956E+04/T(K)                                                    1079
      IF(TEA-80.0)  1,1,2                                                   1080
2     EM(K)=0.0                                                             1081
3     GO TO 5                                                               1082
1     EM(K)=1.0E+05/(EXPF(3.956E+04/T(K))-1.0)                              1083
5     CONTINUE                                                              1084
                                                                            1085
C  OPTICAL DEPTHS                                                           1086
      DO 200  K=1,LZ                                                        1087
      DM(K)=0.0                                                             1088
      DZ(K)=0.0                                                             1089
      DP(K)=(R(K)+R(K-1))/2.0                                               1090
      LS=K                                                                  1091
      DO 100   L=LS,LZ                                                      1092
300   IF(L-K)  320,310,320                                                  1093
310   DZ(L)=SQRTF(R(L)**2-DP(K)**2)                                         1094
311   DM(L)=DZ(L)                                                           1095
313   GO TO 100                                                             1096
320   DZ(L)=SQRTF(R(L)**2-DP(K)**2)-DM(L-1)                                 1097
324   DM(L)=DM(L-1)+DZ(L)                                                   1098
100   HZ(L)=EXPF(-BN(L)*DZ(L)/V(L))                                         1099
C    BRIGHTNESS INTEGRATION                                                 1100
      HP(LZ)=0.0                                                            1101
      LS=LZ-K+1                                                             1102
      DO 7  J=1,LS                                                          1103
      M=LZ-J                                                                1104
7     HP(M)=HP(M+1)*HZ(M+1)+(1.0-HZ(M+1))*EM(M+1)                           1105
      LS=K                                                                  1106
      DO 6  J=LS,LZ                                                         1107
6     HP(J)=HP(J-1)*HZ(J)+(1.0-HZ(J))*EM(J)                                 1108
      HPP(K)=HP(LZ)                                                         1109
      IF(K-3)        155,150,155                                            1110
150   HNORM=HPP(3)                                                          1111
      IF(NMC)   155,151,155                                                 1112
151   SCALE=HPP(3)*10.0                                                     1113
155   CONTINUE                                                              1114
199   HPPP(K)=HPP(K)/SCALE                                                  1115
200   HPP(K)=HPP(K)/HNORM                                                   1116
210   WRITE OUTPUT TAPE  JTAPE,201,NMC,TIME,(K,DP(K),HPPP(K),HPP(K),K=1,    1117
210  1LZ)                                                                   1118
201   FORMAT(1H1,6HPHOTOG,I4,10X,5HTIME=1PE10.3//5H ZONE,3X,6HMEAN R,4X,     1119
201  25HABS B,5X,5HREL B//(I6,1P3E10.3))                                    1120
280   RETURN                                                                1121
      END                                                                   1122
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*     FORTRAN                                                          1123
CF1DIAG                                                                1124
      SUBROUTINE DIAGNS                                                1125
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM     1126
35    FORMAT(1H1,4HNMC=I4,5X,4HNTI=I4,5X,4HKZ2=I5,5X,5HZONE=I4,5X,4HRUN=1127
     2I4/)                                                             1128
      M=KZ5                                                            1129
      IF(KZ2 )   5,10,15                                               1130
5     WRITE OUTPUT TAPE 6,6,KZ2                                        1131
6     FORMAT(1H1,21HDIAG CALLED FROM MAIN15)                           1132
      GO TO 20                                                         1133
10    WRITE OUTPUT TAPE 6,11,KZ2                                       1134
11    FORMAT(1H1,22HDIAG CALLED FROM CGSPOI5)                          1135
      GO TO 20                                                         1136
15    WRITE OUTPUT TAPE 6,16,KZ2                                       1137
16    FORMAT(1H1,22HDIAG CALLED FROM HYDRO15)                          1138
20    WRITE OUTPUT TAPE 6,21,NMC,NTI,LZ,LZR,MCP,MCL,TIME,DT,CS,CR,BR,RS,1139
20   1(K,V(K),VR(K),E(K),ER(K),RESDUE(K),PA(K),DIVFA(K),DIVFR(K),K=1,LZ)1140
21    FORMAT(1H ,4HNMC=I4,2X,4HNTI=I4,2X,3HLZ=I4,2X,4HLZR=I4,2X,4HMCP=I4 1141
21   1,2X,4HMCL=I4/6H TIME=1PE12.4,2X,3HDT=1PE12.4,2X,3HCS=0PF5.2,2X,   1142
21   23HCR=0PF5.2,2X,3HBR=0PF5.2,2X,3HRS=0PF5.2//3H  K,6X,1HV,13X,2HVR, 1143
21   312X,1HE,13X,2HER,12X,6HRESDUE,8X,2HPA,12X,5HDIVFA,9X,5HDIVFR//     1144
21   4(I4,1P8E14.7))                                                   1145
22    WRITE OUTPUT TAPE 6,23,NMC,NTI,DTMIN,YIELD,SCALE,TFLUX,FBR,TIMEW,1146
22   1(K,T(K),DTM(K),DET(K),DPT(K),BN(K),CN(K),DN(K),EN(K),K=1,LZ)      1147
23    FORMAT(1H1,4HNMC=I4,2X,4HNTI=I4,2X,6HDTMIN=1PE12.4,2X,6HYIELD=1PE1 1148
23   12.4,2X,6HSCALE=1PE12.4,2X/7H TFLUX=1PE12.4,2X,4HFBR=1PE12.4,2X,6HT 1149
23   2IMEW=1PE12.4//3H  K,6X,1HT,13X,3HDTM,11X,3HDET,11X,3HDPT,11X,2HBN, 1150
23   312X,2HCN,12X,2HDN,12X,2HEN//(I4,1P8E14.7))                       1151
      WRITE OUTPUT TAPE 6,35,NMC,NTI,KZ2,M,NR                          1152
24    WRITE OUTPUT TAPE 6,25,(K,FOS(K),FIS(K),BB(K,1),BC(K,1),R(K),    1153
24   1ZO(K,1),A(K,1),ZI(K,1),K=1,LZ)                                   1154
25    FORMAT(1H /3H  K,6X,2HFO,12X,2HFI,12X,2HBB,12X,2HBC,12X,1HR,13X, 1155
25   12HZO,12X,1HA,13X,2HZI/(I4,1P8E14.7))                             1156
      WRITE OUTPUT TAPE 6,35,NMC,NTI,KZ2,M,NR                          1157
26    WRITE OUTPUT TAPE 6,27,(K,S(K,1),W(K,1),DTAU(K,1),R2(K),  ZMAS(K),1158
26   1TAU(K,1),CWL(K,1),TR(K),K=1,LZ)                                  1159
27    FORMAT(1H /3H  K,6X,1HS,13X,1HW,13X,4HDTAU,10X,2HR2,12X,4HZMAS,10X 1160
27   1,3HTAU,11X,3HCWL,11X,2HTR/(I4,1P8E14.7))                         1161
      WRITE OUTPUT TAPE 6,35,NMC,NTI,KZ2,M,NR                          1162
28    WRITE OUTPUT TAPE 6,29,(K,DMM(K),DM(K),DZ(K),DP(K),DPP(K),Q(K),  1163
28   1FO(K,2),FI(K,2),K=1,LZ)                                          1164
29    FORMAT(1H /3H  K,6X,3HDMM,11X,2HDM,12X,2HDZ,12X,2HDP,12X,3HDPP,11X 1165
29   1,1HQ,13X,5HFOTJZ,9X,5HFITJZ/(I4,1P8E14.7))                       1166
      WRITE OUTPUT TAPE 6,35,NMC,NTI,KZ2,M,NR                          1167
30    WRITE OUTPUT TAPE 6,31,(K,FMM(K),FM(K),FZ(K),HZ(K),FP(K),HP(K),  1168
30   1HPP(K),HPPP(K),K=1,LZ)                                           1169
31    FORMAT(1H /3H  K,6X,3HFMM,11X,2HFM,12X,2HFZ,12X,2HHZ,12X,2HFP,12X, 1170
31   12HHP,12X,3HHPP,11X,4HHPPP/(I4,1P8E14.7))                         1171
      WRITE OUTPUT TAPE 6,35,NMC,NTI,KZ2,M,NR                          1172
32    WRITE OUTPUT TAPE 6,33,TDN2,TD02,TIMES,Q47,Q71,FLIX,FLOX,FLEX,   1173
```

```
32    1RM,RX,RJ,(K,FO(K,3),DTR(K),RZ(K),P(K),DE(K),RR(K),RHOZ(K), U(K),     1174
32    2K=1,100)                                                              1175
33     FORMAT(1H ,2X,5HTON2=1PE12.4,2X,5HTOO2=1PE12.4,2X,6HTIMES=1PE12.4,    1176
33    12X,4HQ47=1PE12.4,2X,4HQ71=1PE12.4,2X/6H FLIX=1PE12.4,2X,5HFLOX=1PE   1177
33    212.4,2X,5HFLEX=1PE12.4,2X,3HRM=1PE12.4,2X,3HRX=1PE12.4,2X,3HRJ=1PE   1178
33    312.4//3H  K,6X,3HDTH,11X,3HDTR,11X,2HRZ,12X,1HP,13X,2HDE,12X,2HRR,   1179
33    412X,4HRHOZ,10X,2HU /(I4,1P8E14.7))                                    1180
       RETURN                                                                1181
       END                                                                   1182
*      FORTRAN                                                               1183
CF1FLUX                                                                     1184
       SUBROUTINE FLUXS                                                      1185
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM           1186
500    LZM1=LZ-1                                                             1187
       DO 530  N=1,NSZ                                                       1188
C   BOUNDARY CONDITION   NO INWARD FLUX AT OUTER BOUNDARY                   1189
       FI(LZ,N)=0.0                                                          1190
510    DO 512   J=1,LZM1                                                     1191
511    K=LZ-J                                                                1192
512    FI(K,N)=FI(K+1,N)*ZI(K+1,N)+BB(K,1)*(A(K+1,N))-2.0*S(K,N)*W(K+        1193
512   11,N)                                                                  1194
520    FO(1,N)=FI(1,N)*ZI(1,N)+BB(1,2)*(A(1,N))+2.0*S(1,N)*W(1,N)            1195
522    DO 525   K=2,LZ                                                       1196
525    FO(K,N)=FO(K-1,N)*(R2(K-1)/R2(K))*ZO(K,N)                             1197
525   2+BB(K,2)*(A(K,N))+2.0*S(K,N)*W(K,N)                                   1198
525   3+(1.0-(R2(K-1)/R2(K)))*FI(K,N)*ZI(K,N)                               1199
530    CONTINUE                                                              1200
549    DO  555   K=1,LZ                                                      1201
       FIS(K)=0.0                                                            1202
       FOS(K)=0.0                                                            1203
       DO 550  N=1,NSZ                                                       1204
       FIS(K)=FIS(K)+FI(K,N)                                                 1205
550    FOS(K)=FOS(K)+FO(K,N)                                                 1206
551    IF(K-1)   552,552,554                                                 1207
552    D1=0.0                                                                1208
553    GO TO 555                                                             1209
554    D1=1.0                                                                1210
555    DIVFA(K)=(R2(K)/ZMAS(K))*(FOS(K)-FIS(K))                              1211
555   1-D1*(R2(K-1)/ZMAS(K))*(FOS(K-1)-FIS(K-1))                             1212
541    FLOX=12.56*R2(LZ-3)*FOS(LZ-3)                                         1213
590    RETURN                                                                1214
591    END                                                                   1215
                                                                             1216
*      FORTRAN                                                               1217
CF1STE                                                                      1218
       SUBROUTINE STATE                                                      1219
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM           1220
                                                                             1221
C    STEFAN =2.514E-09 TO INCLUDE RADIATION ENERGY AND PRESSURE              1222
C   STEFAN =0.00 TO DELETE RADIATION ENERGY AND PRESSURE                     1223
       STEFAN=0.0                                                            1224
```

```
100    DO 126  K=1,LZ                                                          1225
101    ETA(K)=1.0/(1.293E-03* V(K))                                            1226
102    TS(K)=TS(K)*1.0E-04                                                     1227
103    XS(K)=1.0E+04*TS(K)/( ETA(K)**0.086)                                    1228
104    GNU(K)=LOGF(XS(K)/2000.0)/LOGF(1250.0)                                  1229
105    IF(XS(K)-2000.0) 106,106,108                                            1230
106    PART(K)=1.0                                                             1231
107    GO TO 112                                                               1232
108    IF(XS(K)-2.5E+06)  109,111,111                                          1233
109    PART(K)=1.0+15.4*( GNU(K)**3)*(4.0-3.0* GNU(K))                         1234
110    GO TO 112                                                               1235
111    PART(K)=16.4                                                            1236
112    P(K)= (2.881*(TS(K)/ V(K))* PART(K)+STEFAN*(TS(K)**4))*1.0E+10          1237
113    Y=0.0784/(2.881*TS(K)* PART(K))                                         1238
114    UZ=1.0+(27.0*Y+3.0)/(5.0*Y+1.0)+861.0*(1.0-Y)*Y/(3000.0*(Y**2)+1.0      1239
114   1)+(2356.0*(1.0-Y)*Y)/(9.0E+04*(Y**2)+1.0)+( 41000.0*(1.0-Y)*Y)/         1240
114   2(12.0E+06*(Y**2)+1.0)+(2.15E+05*(1.0-Y)*Y)/(1.5E+18*(Y**4)+1.0)         1241
115    UT=(24.0*(Y**2)+4.0E-10)/(4.*(Y**2)+1.E-10)-(0.0970*(Y**2)*(1.0-Y)      1242
115   1)/(2.0E-06+Y**3)+(4.18E-05*(Y**3)*(1.0-Y))/(1.14E-11+Y**6)              1243
116    UC=UZ-0.09*(UZ-UT)*LOGF( ETA(K))                                        1244
117    E(K)= (0.03920*(UC-1.0)/Y+3.0*STEFAN*(TS(K)**4)* V(K))*1.0E+10          1245
126    TS(K)=TS(K)*1.0E+04                                                     1246
       TS(2)=TS(2)*1.0E-04                                                     1247
       TS(1)=TS(1)*1.0E-04                                                     1248
       P(1)=(0.538E+10*TS(1)**1.5)/V(1)                                        1249
       E(1)=2.53E+10*(((V(1)**0.25)+10.0)/(3.0*V(1)**0.5+10.0))*               1250
      1((V(1)**0.25)+0.13)*(TS(1)**1.5+.02722/V(1)**1.5)                       1251
       P(2)=36.18*TS(2)*1.0E+10*(920.0+TS(2)**2)/(V(2)*(TS(2)**2+1.08E+04      1252
      1))                                                                      1253
       E(2)=((649.0+TS(2)**2)/(100.0+TS(2)))                                   1254
      1*((82.7*TS(2)*1.0E+10)/(116.0/(V(2)**0.25)+TS(2)*(1.0+0.12/(V(2)**     1255
      20.25))))                                                                1256
       TS(1)=TS(1)*1.0E+04                                                     1257
       TS(2)=TS(2)*1.0E+04                                                     1258
176    RETURN                                                                  1259
177    END                                                                     1260
*      FORTRAN                                                                 1261
CF1HYDO                                                                        1262
       SUBROUTINE HYDRO                                                        1263
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM             1264
                                                                               1265
       KHR=0                                                                   1266
       KRS=1                                                                   1267
       KZ4=0                                                                   1268
C    TEST FOR IMPROPER CONVERGENCE                                             1269
       DO 195  K=1,LZ                                                          1270
       IF(DTM(K))     192,195,191                                              1271
191    IF(T(K)-2.0E+05)          188,188,189                                   1272
188    IF(ABSF(DTM(K))-4.99*T(K)) 195,195,193                                  1273
189    IF(ABSF(DTM(K))-0.80*T(K)) 195,195,193                                  1274
192    IF(ABSF(DTM(K))-0.25*T(K)) 195,195,193                                  1275
```

```
193   KZ2=193                                                                  1276
      RQX=RQX*0.2                                                              1277
      RM=RQX*1.3                                                               1278
      CALL DIAGNS                                                              1279
194   GO TO 198                                                                1280
195   CONTINUE                                                                 1281
198   IF(RQX)  199,199,200                                                     1282
199   RQX=DTMIN                                                                1283
C RICHTMYER  -  VON NEUMANN HYDRODYNAMIC SCHEME                                1284
C   ADVANCE VELOCITY - - CHOOSE TIME STEP                                      1285
200   DO 207  K=1,LZ                                                           1286
201   W(K,3)=U(K)                                                              1287
202   UR(K)=U(K)                                                               1288
203   U(K)= U(K)-2.0*((R2(K)    *DT)/(ZMAS(K)+ZMAS(K+1)))*(P(K+1)+Q(K+1)       1289
204  1-P(K)-Q(K))                                                              1290
206   IF(U(K)-1.0E+11) 207,210,210                                             1291
207   CONTINUE                                                                 1292
208   U(LZ)=0.0                                                                1293
209   GO TO 217                                                                1294
210   KZ2=210                                                                  1295
      CALL DIAGNS                                                              1296
213   GO TO 52                                                                 1297
C  HYDRODYNAMIC TIME STEP - - COURANT CRITERION                                1298
217   RM=RQX*1.3                                                               1299
218   IF(RM-0.070*TIME)  222,222,219                                           1300
219   RM=0.070*TIME                                                            1301
222   DO 227  K=1,LZ                                                           1302
224   DTH(K)=DR(K)/(RS*SQRTF(P(K)/(V(K)*(RHOZ(K)**2))))                        1303
      DTH(K)=DTH(K)*((RZ(K)/R(K))**2)                                          1304
      F0(K,3)=DTH(K)                                                           1305
225   IF(RM-DTH(K)) 227,226,226                                                1306
226   RM=DTH(K)                                                                1307
      KZ4=K                                                                    1308
227   CONTINUE                                                                 1309
228   RJ=RM                                                                    1310
                                                                               1311
C    RADIATION LIMITED TIME STEP                                               1312
229   RX=RM                                                                    1313
230   DO 242  K=KRS,LZ                                                         1314
      IF(K-1)  233,233,223                                                     1315
223   IF(R(K)-FBR)  231,231,233                                                1316
231   AR=BR                                                                    1317
232   GO TO 234                                                                1318
233   AR=BR*2.0                                                                1319
234   CONTINUE                                                                 1320
235   DTR(K)=ABSF(AR*E(K)/DIVFA(K))                                            1321
238   IF(DTR(K)-DTMIN  )  242,242,239                                          1322
239   IF(DTR(K)-RX)  240,240,242                                               1323
240   RX=DTR(K)                                                                1324
241   KZ4=K                                                                    1325
242   CONTINUE                                                                 1326
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```
243   RM=RX                                                                    1327
244   IF(RM-2.0*DT)  246,246,245                                               1328
245   RM=2.0*DT                                                                1329
246   IF(RM-DTMIN)  247,248,248                                                1330
247   RM=DTMIN                                                                 1331
248   RQX=RM                                                                   1332
249   IF((TIME+RM)-TIMEW)  252,252,250                                         1333
250   IF(NMC- 4)  252,252,251                                                  1334
251   RM=TIMEW-TIME                                                            1335
252   DO 255  L=1,LZM1                                                         1336
253   UA(L)=(U(L)/2.0)+(UR(L)/2.0)+(U(L)*RM)/(2.0*DT)-(UR(L)*RM)/(2.*DT)       1337
254   UR(L)=(U(L)/2.0)+(UR(L)/2.0)-(U(L)*RM)/(2.0*DT)+(UR(L)*RM)/(2.*DT)       1338
255   U(L)=UA(L)                                                               1339
256   DT=RM                                                                    1340
C  ADVANCE RADIUS ALL L                                                       1341
260   DO 263  L=1,LZ                                                           1342
      W(L,5)=R(L)                                                              1343
261   R(L)=R(L)+DT*U(L)                                                        1344
262   RR(L)=R(L)/RZ(L)                                                         1345
263   R2(L)=R(L)**2                                                            1346
      DO 269  K=2,LZ                                                           1347
      IF((R(K)-R(K-1))-0.15)   264,264,269                                     1348
264   KHR=KHR+1                                                                1349
      IF(KHR-3)  265,265,270                                                   1350
265   KZ2=266                                                                  1351
266   CALL DIAGNS                                                              1352
      DO 268  L=1,LZ                                                           1353
      U(L)=W(L,3)                                                              1354
      R(L)=W(L,5)                                                              1355
268   CONTINUE                                                                 1356
      RQX=DT*0.2                                                               1357
      GO TO 200                                                                1358
269   CONTINUE                                                                 1359
C  ADVANCE SPECIFIC VOLUME                                                     1360
270   V(1)=((R(1)**3)/(RZ(1)**3))/RHOZ(1)                                      1361
271   DO  277  K=2,LZ                                                          1362
273   V(K)=(1.0/RHOZ(K))*(((R(K)/DR(1))**3)-((R(K-1)/DR(1))**3))/              1363
274  1(((RZ(K)/DR(1))**3)-((RZ(K-1)/DR(1))**3))                                1364
275   IF(V(K)-1.0E+20)   276,279,279                                           1365
276   IF(V(K))     279,279,277                                                 1366
277   CONTINUE                                                                 1367
278   GO TO 283                                                                1368
279   KZ2=279                                                                  1369
      CALL DIAGNS                                                              1370
282   GO TO 52                                                                 1371
C  ADVANCE ARTIFICAL VISCOSITY                                                 1372
283   IF(NMC-NQS)   284,284,286                                                1373
284   VD=CS                                                                    1374
285   GO TO 287                                                                1375
286   VD=CR                                                                    1376
287   DO 295  K=1,LZ                                                           1377
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```
      IF(V(K)-VR(K)) 293,2942,2942                                          1378
293   Q(K)=(VD*((ZMAS(K)/R(K)**2)**2)/(V(K)+VR(K)))*(((V(K)-VR(K))/DT)**    1379
     12)                                                                    1380
294   IF(Q(K)-1.0E+22) 2941,298,298                                         1381
2941  IF(     Q(K)) 2942,295,295                                            1382
2942  Q(K)=0.0                                                              1383
295   CONTINUE                                                              1384
296   Q(LZ)=0.0                                                             1385
297   RETURN                                                                1386
298   KZ2=298                                                               1387
      CALL DIAGNS                                                           1388
52    WRITE OUTPUT TAPE 6,43,T(1)                                           1389
43    FORMAT(1H ,15HEXIT FROM HYDRO,5X,5HT(1)=1PE13.6)                      1390
      T(1)=0.000000                                                         1391
1500  GO TO 297                                                             1392
      END                                                                   1393
                                                                            1394
*     FORTRAN                                                               1395
CFICOEF                                                                     1396
      SUBROUTINE  COEFF                                                     1397
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM          1398
700   LZR=LZR                                                               1399
C  SET FLUX DERIVATIVES NEAR OUTER BOUNDARY OF RADIATIVE REGION             1400
701   FP(LZR)=0.0                                                           1401
702   HP(LZR)=0.0                                                           1402
703   HPP(LZR)=0.0                                                          1403
704   HPPP(LZR)=0.0                                                         1404
705   HPP(LZR-1)=0.0                                                        1405
706   HPPP(LZR-1)=0.0                                                       1406
707   HPPP(LZR-2)=0.0                                                       1407
      DO 769  J=1,LZR                                                       1408
C   SET RIPPLE ZONE PARAMETERS                                              1409
      DO  514  N=1,NSZ                                                      1410
      SAVE1=BB(J,N)                                                         1411
      SAVE2=BB(J-1,N)                                                       1412
      SAVE3=S(J,N)                                                          1413
      SAVE4=S(J-1,N)                                                        1414
      SAVE5=DTAU(J,N)                                                       1415
      SAVE6=A(J,N)                                                          1416
      SAVE7=W(J,N)                                                          1417
      SAVE8=ZI(J,N)                                                         1418
      SAVE9=ZI(J-1,N)                                                       1419
      SAVE10=ZO(J,N)                                                        1420
      SAVE11=ZO(J+1,N)                                                      1421
      SAVE12=BC(J,N)                                                        1422
      SAVE14=T(J)                                                           1423
      SAVE13=TS(J)                                                          1424
      SAVE15=CWL(J,1)                                                       1425
      SAVE16=ZO(J+2,N)                                                      1426
      SAVE17=BB(J,2)                                                        1427
      SAVE18=BB(J-1,2)                                                      1428
```

```
311    BB(J,N)=(((((R(J+1)-R(J))*TP(J)**4)+((R(J)-R(J-1))*T (J+1)**4))/     1429
311   1(R(J+1)-R(J-1))))*5.67E-05                                           1430
       BB(J-1,N)=(((((R(J)-R(J-1))*T(J-1)**4)+((R(J-1)-R(J-2))*TP(J)**4))  1431
      1/(R(J)-R(J-2))))*5.67E-05                                           1432
       BB(J-1,2)=BB(J-1,1)                                                 1433
       BB(J,2)=BB(J,1)                                                     1434
312    BC(J,N)=5.67E-05*(TP(J)**4)                                         1435
       TS(J)=TP(J)/11606.5                                                 1436
       T(J)=TP(J)                                                          1437
       ETA(J)=1.293E-03*V(J)                                               1438
       ALPHA=9.0+EXPF(-TIME/1.0E-03*(ETA(J)**2))                           1439
       BETA=((6.0E-18*V(J))**0.5)*(TS(J)**0.1)                             1440
300    ZI(J,1)=EXPF(-1.5*(R(J)-R(J-1))    /((ETA(J)**1.5)/((TS(J)**ALPHA)  1441
300   2+BETA)          +0.2*(ETA(J)**1.91)/(TS(J)**2.73)                   1442
300   3+0.023*(ETA(J)**1.8)*(TS(J)**0.25)+1.0E-07*(ETA(J)**2.0)/          1443
300   4((TS(J)**(-5.0))+4.0E-14*V(J))))                                   1444
       ZI(J,1)=ZI(J,1)*0.99999998                                          1445
       ZI(J,2)=ZI(J,1)                                                     1446
       HMFP=(ETA(J)**1.5)/((TS(J)**ALPHA)+BETA)                            1447
      1+0.2*(ETA(J)**1.91)/(TS(J)**2.73)                                   1448
      2+0.023*(ETA(J)**1.8)*(TS(J)**0.25)                                  1449
      3+1.0E-07*(ETA(J)**2.0)*(TS(J)**5.00)                                1450
       HMFF=9.3E-06*(ETA(J)**2.0)*(TS(J)**3.5)                             1451
       IF(TS(J)-10.0)  301,298,298                                         1452
298    IF(HMFF-HMFP)  299,301,301                                          1453
299    HMFP=HMFF                                                           1454
301    DTAU(J,1)=1.5*(R(J)-R(J-1))/HMFP                                    1455
302    A(J,N)=EXPF(-DTAU(J,N))                                             1456
305    TS(J)=TP(J)                                                         1457
506    S(J,N)=(BC(J,N)-BC(J+1,N))/(DTAU(J,N)+DTAU(J+1,N))                  1458
       S(J-1,N)=(BC(J-1,N)-BC(J,N))/(DTAU(J-1,N)+DTAU(J,N))                1459
507    W(J,N)=1.0-A(J,N)-A(J,N)*DTAU(J,N)                                  1460
508    IF(W(J,N)-1.0E-04)  509,514,514                                     1461
509    W(J,N)=0.5*(DTAU(J,N)**2)                                           1462
514    CONTINUE                                                            1463
                                                                           1464
C    SET CUTOFF WAVELENGTHS                                                1465
540    I=J                                                                 1466
541    IF(T(I)-TDN2)  546,542,542                                          1467
                                                                           1468
C    OPACITY DUE TO ATOMIC SPECIES                                         1469
542    CWL(I,1)= 700.0*EXPF(-0.36*(T(I)-11606.5)/11606.5)                  1470
       IF(CWL(I,1)-275.0)   545,560,560                                    1471
545    CWL(I,1)=275.0                                                      1472
       GO TO 560                                                           1473
546    IF(T(I)-TD02)  551,547,547                                          1474
                                                                           1475
C  OPACITY DUE TO N2 MOLECULE                                              1476
547    TCN2=5.0E-07*1.293E-03*V(I)                                         1477
       CWL(I,1)=1140.0*(((R(I)-R(I-1))/(V(I)*1.293E-03))**0.11)            1478
      1*(1.0-EXPF(-TIME/TCN2))                                             1479
```

```
      IF(CWL(I,1)-1000.0)  550,560,560                                  1480
550   CWL(I,1)=1000.0                                                   1481
      GO TO 560                                                         1482
                                                                        1483
C     OPACITY DUE TO O2 MOLECULE                                        1484
551   TC=4250.0-271.0*LOGF(1.293E-03* V(I))                             1485
      TCO2=3.0E-07*1.293E-03*V(I)                                       1486
553   IF(T(I)-TC)554,554,556                                            1487
554   CWL(I,1)=1500.0+TS(I)*(0.163+0.0745*LOGF(DR(I)*RHOZ(I)/1.293E-03))1488
554  1*(1.0-EXPF(-TIME/TCO2))                                           1489
555   GO TO 557                                                         1490
556   TSB=TS(I)*(.0647*LOGF(R(I)-R(I-1))-0.25-.109*LOGF(1.293E-03*V(I)))1491
      CWL(I,1)=3500.0+TSB-(2000.0+TSB)*EXPF(-TIME/TCO2)                 1492
557   IF(CWL(I,1)-1500.0) 558,560,560                                   1493
558   CWL(I,1)=1500.0                                                   1494
560   CONTINUE                                                          1495
                                                                        1496
C     CALCULATE ZO VALUES                                               1497
      ZO(1,1)=ZI(1,1)                                                   1498
604   JSTR=J+2                                                          1499
      KD=0                                                              1500
605   DO 670   L=J,JSTR                                                 1501
      TR(L)=T(L)                                                        1502
      IF(T(L)-8.0E+04)  607,668,668                                     1503
607   KD=KD+1                                                           1504
      IF(KD-1)  608,608,609                                             1505
608   MS=L-1                                                            1506
609   IF(DTAU(L,1)-1.0)  610,668,668                                    1507
C     SELECT TR(L)= TEMPERATURE OF RADIATION TRAVERSING ZONE L          1508
610   LIM=L-1                                                           1509
611   TAUSUM=0.0                                                        1510
615   DO 630 NN=1,LIM                                                   1511
616   M=L-NN                                                            1512
617   TAUSUM=TAUSUM+DTAU(M,1)                                           1513
620   IF(TAUSUM-0.7)  630,627,627                                       1514
627   TR(L)=T(M)                                                        1515
628   IF(M-2)  630,630,638                                              1516
630   CONTINUE                                                          1517
      IF(T(L)-2500.0)  631,632,632                                      1518
631   TR(L)=TR(L-1)                                                     1519
      GO TO 637                                                         1520
632   EPTSUM=0.0                                                        1521
      EPSUM=0.0                                                         1522
      DO 635  I=3,LIM                                                   1523
      EPTSUM=EPTSUM+T(I)*DTAU(I,1)                                      1524
      EPSUM=EPSUM+DTAU(I,1)                                             1525
      IF(T(I)-2500.0)   636,636,635                                     1526
635   CONTINUE                                                          1527
636   TR(L)=EPTSUM/EPSUM                                                1528
637   M=2                                                               1529
638   IZ=M+1                                                            1530
```

```
639    DO 646    I=IZ,L                                                        1531
640    IF(TR(L)*CWL(I,1)-2.0E+08)  641,641,644                               1532
641    ZI(I,2)= EXPF((-3.41E-08*CWL(I,1)*TR( L )+2.5E-25*((CWL(I,1)*TR(L    1533
641   2))**3))*EXPF(-3.07E+13/((CWL(I,1)*TR( L ))**1.8)))                    1534
642    GO TO 646                                                             1535
644    ZI(I,2)=7.61E+22/((CWL(I,1)*TR( L ))**3)                              1536
646    CONTINUE                                                              1537
       ZMAX=ZI(M+1,2)                                                        1538
       ZO(L,1)=ZI(M+1,2)                                                     1539
647    IF(L-M-1) 668,663,650                                                 1540
C    CORRECTION FOR IMTERMEDIATE ZONES                                       1541
650    IZ=M+2                                                                1542
651    CWLTST=CWL(M+1,1)                                                     1543
652    DO 660  NZ=IZ,L                                                       1544
653    IF(CWL(NZ,1)-CWLTST)  654,654,657                                     1545
654    IF(NZ-L)  660,655,668                                                 1546
655    ZO(L,1)=1.0                                                           1547
656    GO TO 660                                                             1548
657    ZO(L,1)=ZI(NZ,2)/ZMAX                                                 1549
658    CWLTST=CWL(NZ,1)                                                      1550
659    ZMAX=ZI(NZ,2)                                                         1551
660    CONTINUE                                                              1552
663    IF(ZO(L,1)-ZI(L,1))  670,668,668                                      1553
668    ZO(L,1)=ZI(L,1)                                                       1554
670    CONTINUE                                                              1555
671    IF(DTAU(J,1)-1.0E-04)  672,674,674                                    1556
672    A(J,1)=DTAU(J,1)                                                      1557
673    GO TO 675                                                             1558
674    A(J,1)=1.0-A(J,1)                                                     1559
675    IF(S(J,1)-1.69E+38)   8675,8675,677                                   1560
8675   IF((BB(J,1)*A(J+1,1)-2.0*S(J,1)*W(J+1,1))-BC(J+1,1)*A(J+1,1)*0.01)    1561
8675 1   677,680,680                                                         1562
677    S(J,1)=0.0                                                            1563
678    BB(J,1)=BC(J+1,1)                                                     1564
679    BB(J,2)=BC(J,1)                                                       1565
680    IF(S(J-1,1)-1.69E+38)  8680,8680,681                                  1566
8680   IF((BB(J-1,1)*A(J,1)-2.0*S(J-1,1)*W(J,1))-BC(J,1)*A(J,1)*0.01)       1567
8680 1     681,684,684                                                       1568
681    S(J-1,1)=0.0                                                          1569
682    BB(J-1,1)=BC(J,1)                                                     1570
683    BB(J-1,2)=BC(J-1,1)                                                   1571
684    IF(J-2)  900,900,916                                                  1572
900    IF(BC(1,1)-BC(2,1))   915,916,916                                     1573
915    BB(1,2)=BC(1,1)                                                       1574
916    T(J)=SAVE14                                                           1575
       CWL(J,1)=SAVE15                                                       1576
C    CALCULATE TEMPORARY FLUXS                                               1577
831    JSTR=J+2                                                              1578
832    IF(JSTR-LZR)  834,834,833                                             1579
833    JSTR=LZR                                                              1580
834    DO  842 N=1,NSZ                                                       1581
```

```
835    FIT(JSTR+1,N)=FI(JSTR+1,N)                                          1582
836    DO 838  L=1,JSTR                                                    1583
837    K=JSTR-L+1                                                          1584
838    FIT(K,N)=FIT(K+1,N)* ZI(K+1,N)+BB(K,1)*( A(K+1,N))-2.0* S(K,N)      1585
838   2* W(K+1,N)                                                          1586
839    FOT(1,N)=FIT(1,N)* ZI(1,N)+BB(1,2)*( A(1,N))+2.0* S(1,N)            1587
839   2* W(1,N)                                                            1588
840    DO 841   K=2,JSTR                                                   1589
841    FOT(K,N)=FOT(K-1,N)*(R2(K-1)/R2(K))* ZO(K,N)                        1590
841   2+BB(K,2)*( A(K,N))+2.0* S(K,N)* W(K,N)                              1591
841   3+(1.0-(R2(K-1)/R2(K)))*FIT(K,N)* ZI(K,N)                            1592
842    CONTINUE                                                            1593
C   SPECTRAL SUMMATION                                                    1594
726    FOTJP2=0.0                                                          1595
727    FOTJP1=0.0                                                          1596
728    FOTJZ=0.0                                                           1597
729    FOTJM1=0.0                                                          1598
730    FITJZ=0.0                                                           1599
731    FITJM1=0.0                                                          1600
732    FITJM2=0.0                                                          1601
733    FITJM3=0.0                                                          1602
734    DO 742   N=1,NSZ                                                    1603
735    FOTJP2=FOTJP2 + FOT(J+2,N)                                          1604
736    FOTJP1=FOTJP1 + FOT(J+1,N)                                          1605
737    FOTJZ=FOTJZ + FOT(J,N)                                              1606
738    FOTJM1=FOTJM1 + FOT(J-1,N)                                          1607
739    FITJZ=FITJZ + FIT(J,N)                                              1608
740    FITJM1=FITJM1 + FIT(J-1,N)                                          1609
741    FITJM2=FITJM2 + FIT(J-2,N)                                          1610
742    FITJM3=FITJM3 + FIT(J-3,N)                                          1611
       FO(J,2)=FOTJZ                                                       1612
       FI(J,2)=FITJZ                                                       1613
C   CALCULATE FLUX DERIVATIVES                                            1614
743    IF(J-LZR+1)   744,745,746                                           1615
744    FMM(J+2)=(FOTJP2 -FOS(J+2))/(TP(J)-T(J))                            1616
745    FM(J+1)=(FOTJP1 -FOS(J+1))/(TP(J)-T(J))                             1617
746    FZ(J)=(FOTJZ-FOS(J))/(TP(J)-T(J))                                   1618
747    HZ(J)=(FITJZ-FIS(J))/(TP(J)-T(J))                                   1619
748    IF(J-1)  755,755,749                                                1620
749    FP(J-1)=(FOTJM1 -FOS(J-1))/(TP(J)-T(J))                             1621
750    HP(J-1)=(FITJM1-FIS(J-1))/(TP(J)-T(J))                              1622
751    IF(J-2)  755,755,752                                                1623
752    HPP(J-2)=(FITJM2-FIS(J-2))/(TP(J)-T(J))                             1624
753    IF(J-3)  755,755,754                                                1625
754    HPPP(J-3)=(FITJM3-FIS(J-3))/(TP(J)-T(J))                            1626
C   RETURN RIPPLE ZONE PARAMETERS TO NORMAL VALUES                        1627
755    DO 768  N=1,NSZ                                                     1628
       BB(J,N)=SAVE1                                                       1629
       BB(J-1,N)=SAVE2                                                     1630
       S(J,N)=SAVE3                                                        1631
       S(J-1,N)=SAVE4                                                      1632
```

```
      DTAU(J,N)=SAVE5                                                    1633
      A(J,N)=SAVE6                                                       1634
      W(J,N)=SAVE7                                                       1635
      ZI(J,N)=SAVE8                                                      1636
      ZI(J-1,N)=SAVE9                                                    1637
      ZO(J,N)=SAVE10                                                     1638
      ZO(J+1,N)=SAVE11                                                   1639
      BC(J,N)=SAVE12                                                     1640
      TS(J)=SAVE13                                                       1641
      ZO(J+2,N)=SAVE16                                                   1642
      BB(J,2)=SAVE17                                                     1643
      BB(J-1,2)=SAVE18                                                   1644
768   CONTINUE                                                           1645
769   CONTINUE                                                           1646
C  GENERALIZED COEFFICIENT ROUTINE   DEC 6,1963                          1647
770   DO 789   K=1,LZR                                                   1648
771   IF(K-1)  772,772,775                                               1649
772   DMM(1)=0.0                                                         1650
      D1=0.0                                                             1651
773   DM(1)=0.0                                                          1652
774   GO TO 780                                                          1653
775   D1=1.0                                                             1654
      IF(K-2)  777,776,778                                               1655
776   DMM(2)=0.0                                                         1656
777   GO TO 779                                                          1657
778   DMM(K)=(0.5*R2(K)/ZMAS(K))*FMM(K)                                  1658
778  1-D1*(0.5*R2(K-1)/ZMAS(K))*FM(K-1)                                  1659
779   DM(K)= (0.5*R2(K)/ZMAS(K))*FM(K)                                   1660
779  1-D1*(0.5*R2(K-1)/ZMAS(K))*(FZ(K-1)-HZ(K-1))                        1661
780   DZ(K)= (0.5*R2(K)/ZMAS(K))*(FZ(K)-HZ(K))                           1662
780  1-D1*(0.5*R2(K-1)/ZMAS(K))*(FP(K-1)-HP(K-1))                        1663
780  2+(DET(K)/DT)+(DPT(K)/(2.*DT))*(V(K)-VR(K))                         1664
781   DP(K)= (0.5*R2(K)/ZMAS(K))*(FP(K)-HP(K))                           1665
781  1-D1*(0.5*R2(K-1)/ZMAS(K))*(-HPP(K))                                1666
782   DPP(K)=(0.5*R2(K)/ZMAS(K))*(-HPP(K))                               1667
782  1-D1*(0.5*R2(K-1)/ZMAS(K))*(-HPPP(K-1))                             1668
789   CONTINUE                                                           1669
790   DPP(LZR-1)=0.0                                                     1670
791   DP(LZR)=0.0                                                        1671
792   DPP(LZR)=0.0                                                       1672
C  BLOCK 800 SOLUTION OF MATRIX                                          1673
800   CN(1)=DP(1)/DZ(1)                                                  1674
801   DN(1)=DPP(1)/DZ(1)                                                 1675
802   EN(1)=-RESDUE(1)/DZ(1)                                             1676
803   BN(2)=DZ(2)-DM(2)*CN(1)                                            1677
804   CN(2)=(DP(2)-DM(2)*DN(1))/BN(2)                                    1678
805   DN(2)=DPP(2)/BN(2)                                                 1679
806   EN(2)=-(RESDUE(2)+DM(2)*EN(1))/BN(2)                               1680
807   DO 811   K=3,LZR                                                   1681
808   BN(K)=(DZ(K)-DM(K)*CN(K-1)-DMM(K)*DN(K-2)+DMM(K)*CN(K-2)*CN(K-1))  1682
809   CN(K)=(DP(K)-DM(K)*DN(K-1)+DMM(K)*CN(K-2)*DN(K-1))/BN(K)           1683
```

```
810    DN(K)=DPP(K)/BN(K)                                                    1684
811    EN(K)=-(RESDUE(K)+DM(K)*EN(K-1)-DMM(K)*CN(K-2)*EN(K-1)+DMM(K)*EN(K    1685
811   1-2))/BN(K)                                                            1686
812    DN(LZR-1)=0.0                                                         1687
813    BN(LZR)=DM(LZR)-DMM(LZR)*CN(LZR-2)                                    1688
814    CN(LZR)=(DZ(LZR)-DMM(LZR)*DN(LZR-2))/BN(LZR)                          1689
815    DN(LZR)=0.0                                                           1690
816    EN(LZR)=-(RESDUE(LZR)+DMM(LZR)*EN(LZR-2))/BN(LZR)                     1691
819    RETURN                                                                1692
820    END                                                                   1693
C      VERSION C ENTRY ROUTINE                                               1694
       STARTING MODEL IS READ IN FROM DATA CARDS GENERATED                   1695
       BY THE X-RAY DEPOSIT ROUTINE ON UNIVAC 1107                           1696
*      FORTRAN                                                               1697
       SUBROUTINE ENTRY                                                      1698
C     STANDARD DIMENSION AND COMMON STATEMENTS  AS IN MAIN PROGRAM           1699
2      D(1)=0.0                                                              1700
       TEE=3.0E+04                                                           1701
       READ INPUT TAPE 5, 9277,RUNID1,RUNID2,TR(NN1),TR(NN2),ENERGY,TIME,    1702
      1FX,(K,R(K),DR(K),T(K),RHOZ(K),U(K),E(K),RUNID1,RUNID2,K=1,100)        1703
9277   FORMAT(2A6,1P5E13.3/(I4,1P6E11.3,2A5))                                1704
       LZ=100                                                                1705
       VD=CS                                                                 1706
       WKT=ENERGY/4.18E+19                                                   1707
       TDO2=7100.0*EXPF((0.43429*LOGF(RHOZ(99)/1.293E-03))/5.3)              1708
       TDN2=15000.0*EXPF((0.43429*LOGF(RHOZ(99)/1.293E-03))/4.8)             1709
 1019  DO 1033 K=1,100                                                       1710
 1026  V(K)=1.0/RHOZ(K)                                                      1711
       VR(K)=V(K)                                                            1712
 1027  L=K                                                                   1713
       TS(K)=T(K)                                                            1714
       R2(L)=R(L)**2                                                         1715
 1029  RZ(L)=R(L)                                                            1716
 1030  RR(L)=1.0                                                             1717
       TR(K)=T(K)                                                            1718
       ZMAS(K)=(RHOZ(K)/3.0)*((RZ(L)**3)-(RZ(L-1)**3))                       1719
       DU=U(L)-U(L-1)                                                        1720
       Q(K)=-(VD/(2.0*V(K)))*DU*ABSF(DU)                                     1721
       Q(1)=-(VD/(2.0*V(1)))*U(1)*ABSF(U(1))                                 1722
       IF(Q(K))    2233.1033.1033                                            1723
2233   Q(K)=0.0                                                              1724
1033   CONTINUE                                                              1725
       ZMAS(1)=RHOZ(1)*(R(1)**3)/3.0                                         1726
 1034  WRITE OUTPUT TAPE 6,1035,TDN2,TDO2,RN2,RO2,WKT  ,(K,R(K),RHOZ(K),    1727
1034  1V(K),T(K),U(K),Q(K),ZMAS(K),DR(K),DPP(K),DTAU(K,1),TAU(K,1),K=1,      1728
      1100)                                                                  1729
 1035  FORMAT(1H ,10HINPUT DATA,10X,5HTDN2=0PF7.1,10X,5HTDO2=0PF6.1,5X,      1730
1035  14HRN2=1PE9.2,5X,4HRO2=1PE9.2,5X,4HWKT=0PF7.2//4H  K ,2X,4HR(L)        1731
      1    ,7X,7HRHOZ(K),4X,4HV(K),7X,4HT(K),7X,4HU(L),7X,4HQ(K),7X,7HZMAS   1732
      1(K),4X,5HDR(K),7X,6HENG(K),5X,7HDTAU(K),4X,3HTAU//(I4,1P11E11.4))     1733
       T(101)=T(100)                                                         1734
1000   RETURN                                                                1735
       END                                                                   1736
```

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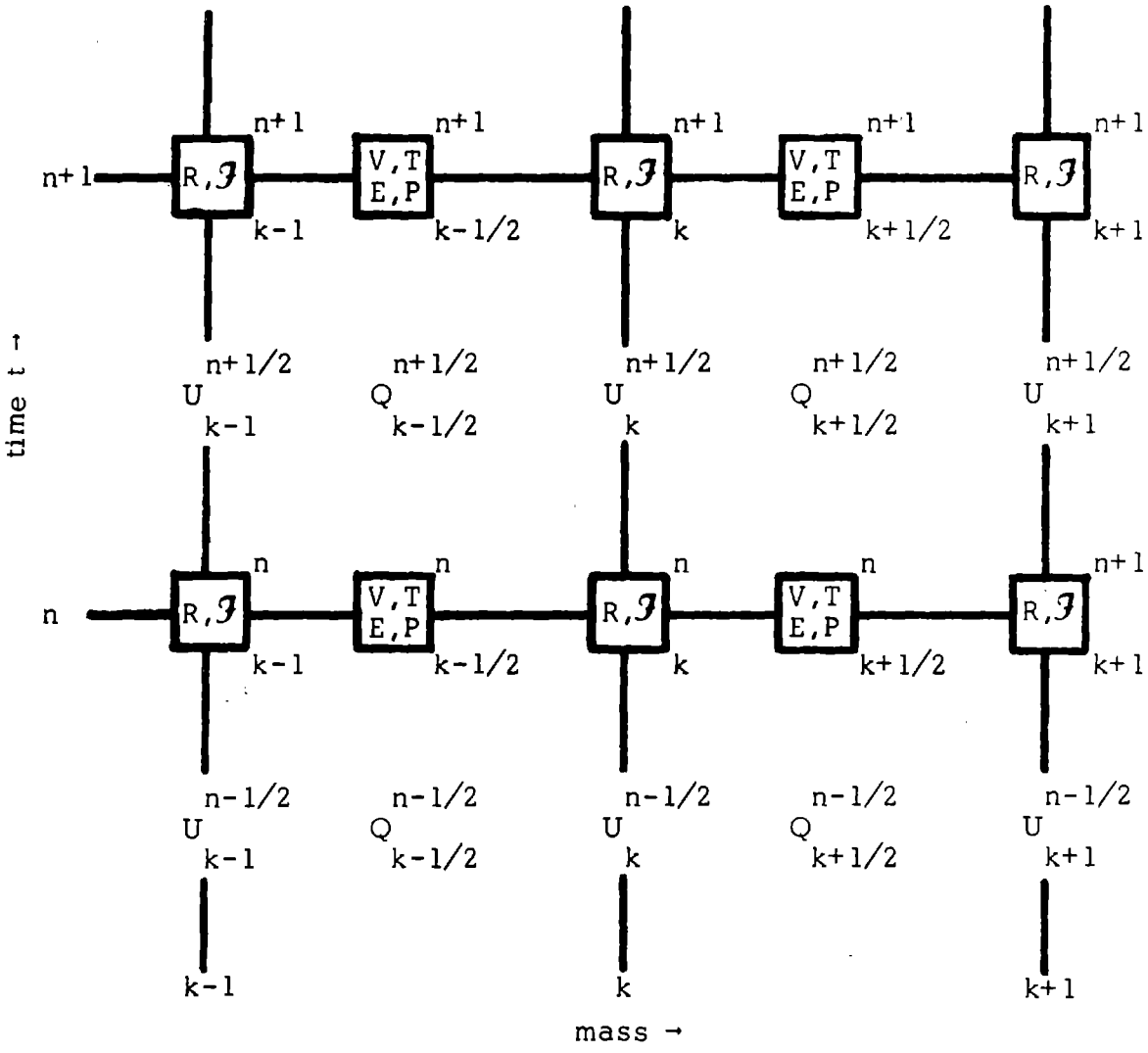

FIG. A-1 FINITE DIFFERENCE GRID

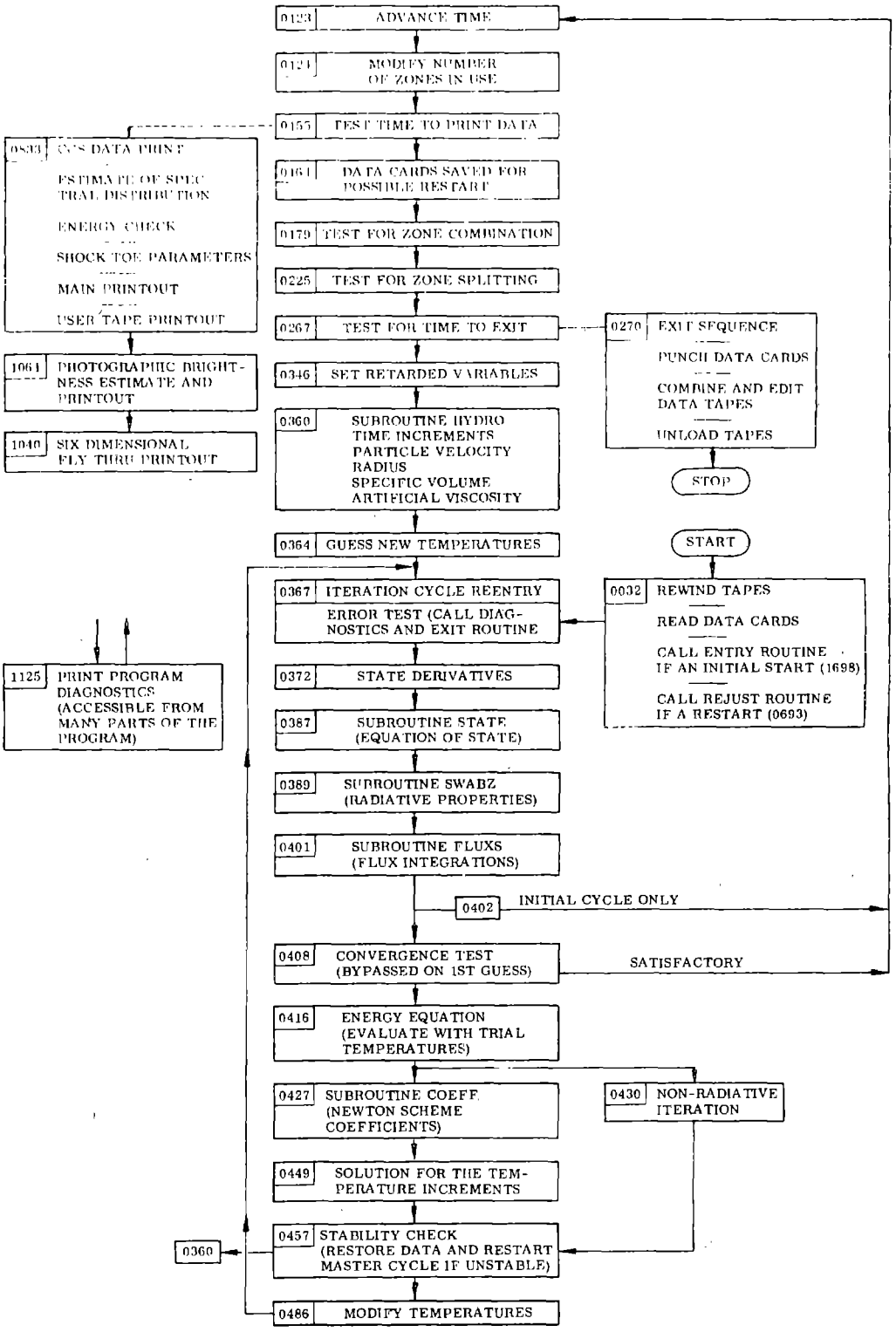

FIG. A-2 SCHEMATIC FLOW CHART

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13. ABSTRACT

This report introduces the reader to radiation hydrodynamics (RH) and discusses its application to fireballs in the atmosphere. After formulating the basic equations of RH, special attention is given to the radiative transfer problem. Several methods for solving the equations of transfer are touched upon but special emphasis is placed on the two stream method with a frequency averaging procedure, which is specifically designed for use with finite zone sizes. A version of the FIREBALL code which utilizes this approach is described. The physics of fireballs is illustrated with the example of a one kiloton detonation at sea level density and without interference from the ground. Some remarks are made on scaling procedures for extending the result to higher yields and altitudes. Estimates are made of the validity of the models.

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